Ph.D. Defense
Adam Pound
DATE: Tuesday, April 6th, 2010
TIME: 10:00 a.m.
PLACE: MacNaughton Room 222
University of Guelph

THESIS TITLE:
Motion of small bodies in general relativity: foundations and implementations of the self-force

ABSTRACT:

Extreme mass-ratio inspirals, in which solar-mass compact bodies spiral into supermassive black holes, are an important potential source for gravitational wave detectors. Because of the extreme mass-ratio, one can treat this problem using perturbation theory. However, in order to relate the motion of the small body to the emitted waveform, one requires a model that is accurate on extremely long timescales. Additionally, in order to avoid intractable divergences, one requires a model that treats the small body as asymptotically small rather than exactly pointlike. Both of these difficulties can be resolved by using techniques of singular perturbation theory. I begin this dissertation with an analysis of singular perturbation theory on manifolds, including the common techniques of matched asymptotic expansions and two-timescale expansions. I then formulate a systematic asymptotic expansion in which the metric perturbation due to the body is expanded while a representative worldline is held fixed, and I contrast it with a regular expansion in which both the metric and the worldline must be expanded. This results in an approximation that is potentially uniformly accurate on long timescales. I illustrate the utility of the expansion for an exact point particle; I then generalize it to an asymptotically small body. The equation of motion for the body's fixed worldline is determined by performing a local-in-space expansion in the neighbourhood of the body. Using this local expansion as boundary data, I then construct a global solution to the perturbative Einstein equations. As a means of concretely characterizing particular orbits, I next devise a relativistic generalization of the Newtonian method of osculating orbits. However, the equations of motion for the body and its metric perturbation are purely formal; a concrete calculation of a particular orbit and waveform brings further difficulties. In order to sidestep these difficulties, earlier authors have suggested making use of adiabatic approximations that capture the dissipative dynamics of the system while ignoring the conservative dynamics. I test the utility of some such approximations in two test cases, making use of the method of osculating orbits and two-timescale expansions.

EXAMINING COMMITTEE:
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