Tidal heating of black holes

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Outline

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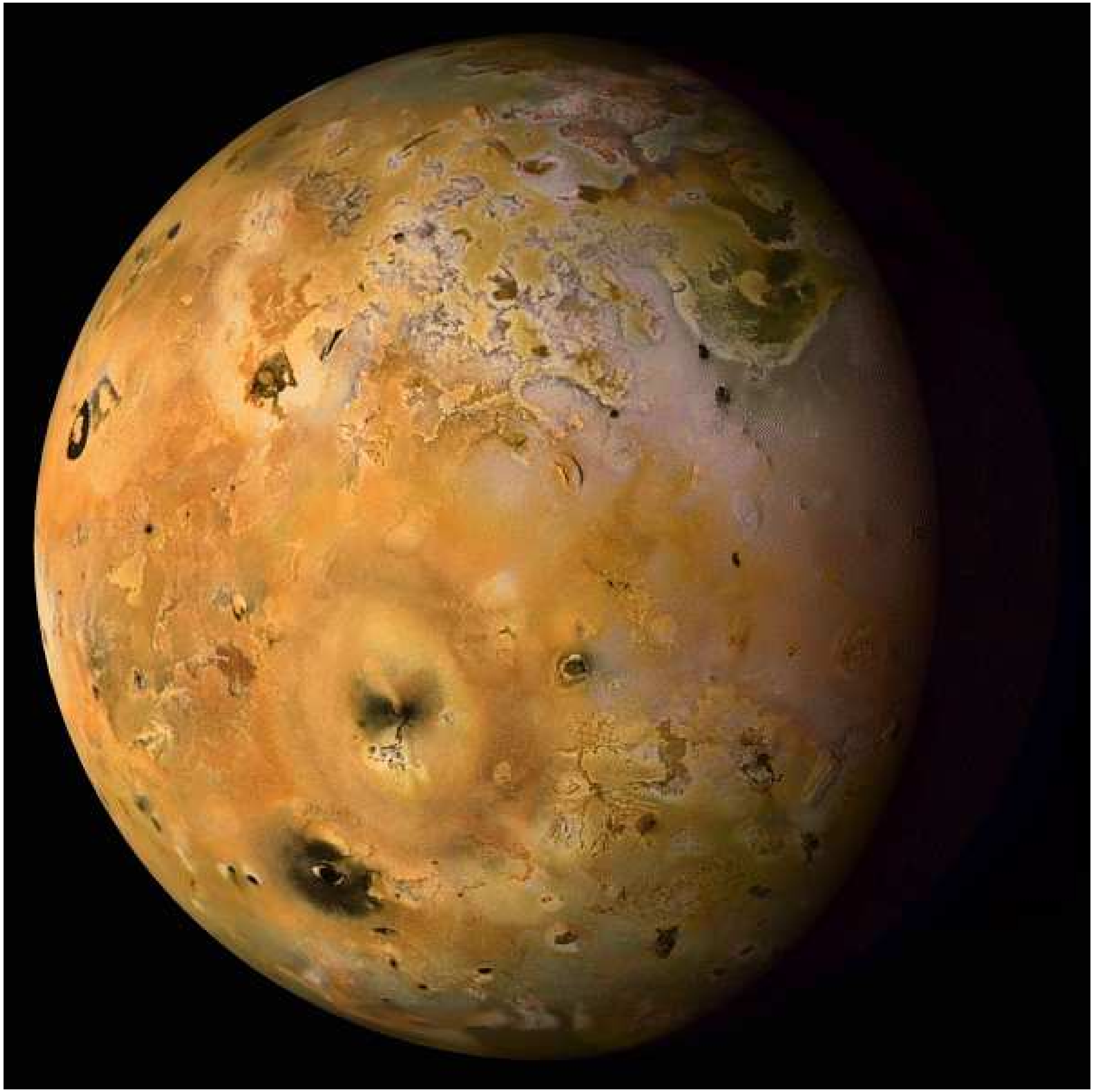
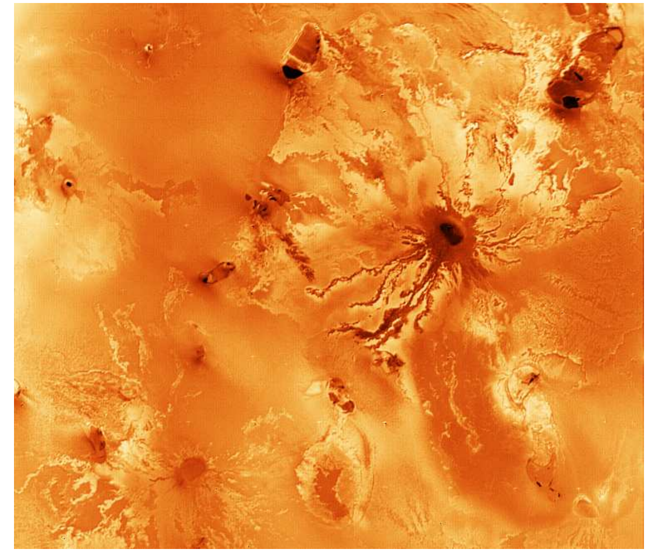
- Tidal heating of a black hole
- So what?
- Conclusion

[Phys. Rev. D **70**, 08044 (2004); Phys. Rev. Lett. **94**, 161103 (2005)]

Tidal heating of Io — Introduction





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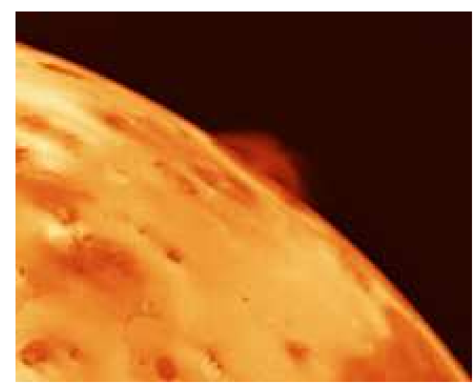


[http://www.solarviews.com/eng/io.htm]

Tidal heating of Io — Introduction

The tidal forces exerted by Jupiter on its satellite Io do work.

This supply of energy goes into heat, which keeps Io’s interior partially melted.

This leads to Io’s observed volcanic activity.

The effect was predicted before its discovery by Voyager I.

[Peale, Cassen, and Reynolds, Science **203**, 892 (1979)]

Tidal heating of Io — Tidal field

Jupiter's tidal gravitational field is described by

$$\mathcal{E}_{ab}(t) = \frac{\partial^2}{\partial x_a \partial x_b} \Phi(t, \boldsymbol{r}_{\rm io})$$

$\Phi(t, \boldsymbol{r}) =$ Jupiter's gravitational potential

$\boldsymbol{g} = -\boldsymbol{\nabla}\Phi =$ Jupiter's gravitational field

Tidal heating of Io — Tidal deformation

The tidal field produces a deformation in an otherwise spherical Io, which acquires a quadrupole moment

$$\begin{aligned} Q_{ab}(t) &= \int \rho\Big(x_a x_b - \frac{1}{3}\delta_{ab} r^2\Big)\, dV \\ &= -\frac{R^5}{2G}\mathcal{E}_{ab}(t-\tau) \qquad\qquad (R = \text{Io's radius}) \end{aligned}$$

where

$$\tau = \frac{19}{2}\frac{R\nu}{GM} \sim 1 \text{ hour} \qquad\qquad (\nu = \text{kinematical viscosity})$$

is the tidal delay $\ll$ orbital period (2 days).

(Here Io is modelled as a spherical mass of viscous, incompressible fluid.)

Tidal heating of Io — Work done

The rate at which the tidal forces do work on Io is given by

$$
\begin{aligned}
\frac{đW}{dt} &= \int \underbrace{(-\rho \mathcal{E}_{ab} x_b)}_{\text{tidal force density}} \; v_a \, dV \\
&= -\frac{1}{2}\dot{Q}_{ab}\mathcal{E}_{ab} \\
&= \frac{R^5}{4G}\dot{\mathcal{E}}_{ab}(t-\tau)\mathcal{E}_{ab}(t) \\
&= \frac{R^5}{4G}(\dot{\mathcal{E}}_{ab} - \tau\ddot{\mathcal{E}}_{ab} + \cdots)\mathcal{E}_{ab} \\
&= -\frac{R^5\tau}{4G}\ddot{\mathcal{E}}_{ab}\mathcal{E}_{ab} + \frac{d}{dt}\left(\frac{R^5}{8G}\mathcal{E}_{ab}\mathcal{E}_{ab}\right) \\
&= \frac{R^5\tau}{4G}\dot{\mathcal{E}}_{ab}\dot{\mathcal{E}}_{ab} + \frac{d}{dt}\left(-\frac{R^5\tau}{4G}\dot{\mathcal{E}}_{ab}\mathcal{E}_{ab} + \frac{R^5}{8G}\mathcal{E}_{ab}\mathcal{E}_{ab}\right)
\end{aligned}
$$

Tidal heating of Io — Heat produced

The total time derivative represents a change of state function, and the rate of work done is

$$\frac{đW}{dt} = \frac{R^5\tau}{4G}\dot{\mathcal{E}}_{ab}\dot{\mathcal{E}}_{ab}$$

This work is converted into heat by Io's viscosity,

$$\frac{đW}{dt} = \frac{đQ}{dt} = \frac{1}{2}\nu \int \rho\, \sigma_{ab}\sigma_{ab}\, dV$$

$$\sigma_{ab} = \frac{\partial v_a}{\partial x_b} + \frac{\partial v_b}{\partial x_a} = \text{shear tensor}$$

This amounts to $\sim 10^{13}$ W, enough to partially melt Io's interior and power the volcanos.

Tidal heating of a BH — BH mechanics

Tidal heating applies to black holes as well as conventional astronomical bodies.

The first law of black-hole mechanics states

$$d(Mc^2) = \underbrace{\frac{c^2}{8\pi G}\, g\, dA}_{\text{heat term}} + \underbrace{\Omega\, dJ}_{\text{work term}}$$

$M =$ mass $\quad g =$ surface gravity $\quad A =$ surface area

$\Omega =$ angular velocity $\qquad J =$ angular momentum

The second law of black-hole mechanics states

$dA \geq 0$ (Hawking's area theorem)

Tidal heating of a BH — BH mechanics

The Hawking process reveals that the hole behaves as a thermodynamic body with temperature

$$T = \frac{\hbar}{2\pi c k_{\rm B}}\, g$$

and entropy

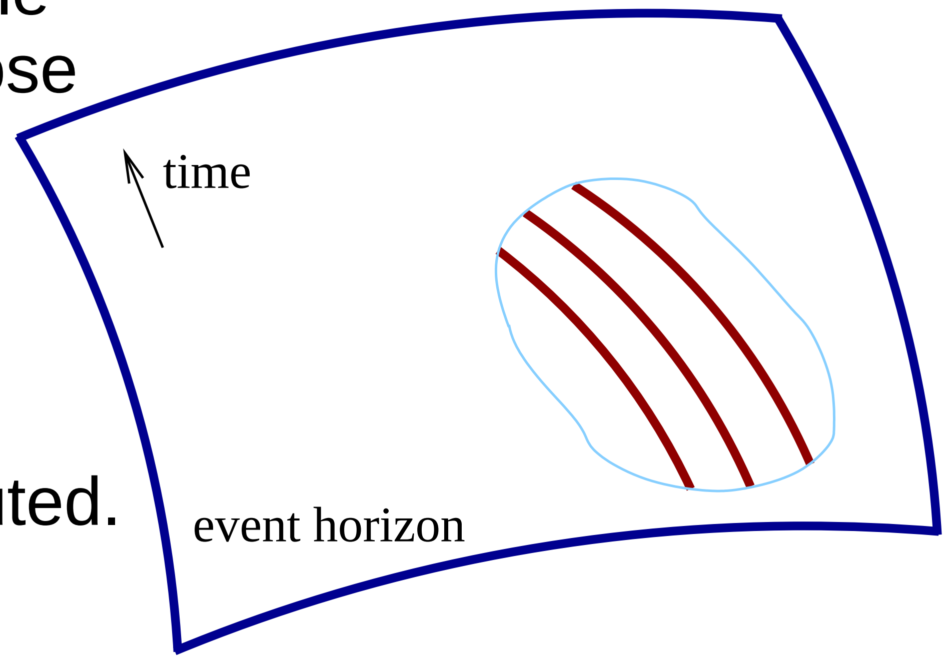
$$S = \frac{c^3 k_{\rm B}}{4G\hbar}\, A$$

The tidal heating of a black hole is therefore calculated by evaluating

$$đQ = T dS = \frac{c^2}{8\pi G}\, g\, dA$$

Tidal heating of a BH — Horizon generators

The event horizon of a black hole is generated by light rays — those rays which are trying to escape but cannot.



The generators of an isolated black hole are uniformly distributed.

But an applied tidal field disturbs the generators — they undergo shearing.

This forces the generators to space themselves out.

The horizon grows in size.

Tidal heating of a BH — Horizon growth

This is described by the *Hawking-Hartle formula*,

[Hawking and Hartle, Commun. Math. Phys. **27**, 283 (1972)]

$$\frac{đQ}{dt} = \frac{c^2}{8\pi G}\, g\, \frac{dA}{dt} = \frac{c^3}{8\pi G} \oint_{\text{horizon}} \sigma_{jk}\sigma^{jk}\, dA$$

$\sigma_{jk} =$ generators' shear tensor

Evaluating this for a tidally distorted black hole gives

$$\boxed{\frac{đQ}{dt} = \frac{1}{180}\frac{R^6}{Gc}\dot{\mathcal{E}}_{ab}\dot{\mathcal{E}}_{ab}}$$

$R = \dfrac{2GM}{c^2} =$ Schwarzschild radius

Tidal heating of a BH — Comparison with Io

This can be compared with the expression that applies to a mass of viscous fluid:

$$\frac{đQ}{dt} \sim \frac{R^6}{Gc}\dot{\mathcal{E}}_{ab}\dot{\mathcal{E}}_{ab} \qquad \text{(black hole)}$$

$$\sim \frac{R^5\tau}{G}\dot{\mathcal{E}}_{ab}\dot{\mathcal{E}}_{ab} \qquad \text{(viscous fluid)}$$

where $\tau \sim R\nu/GM$.

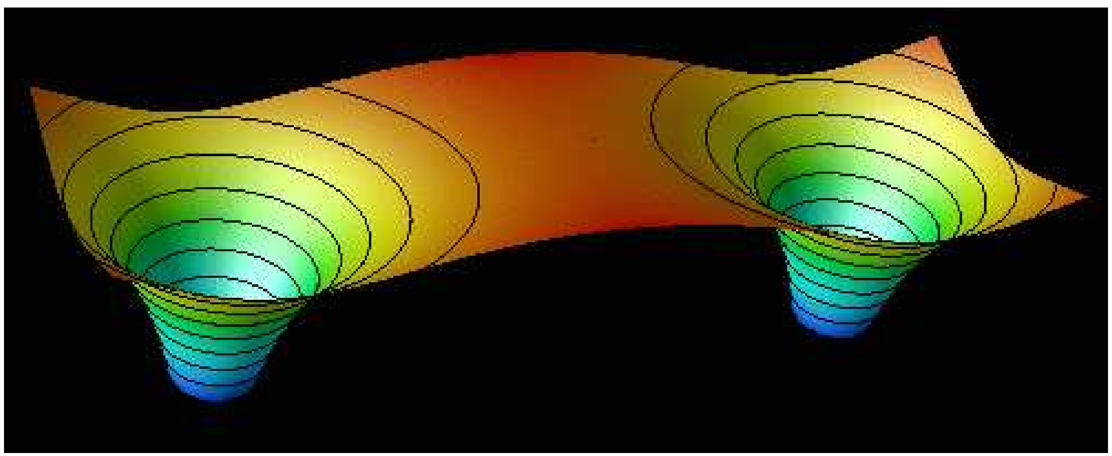
The black-hole tidal delay is thus $\tau \sim R/c$ and the horizon's effective viscosity is $\nu \sim GM/c$.

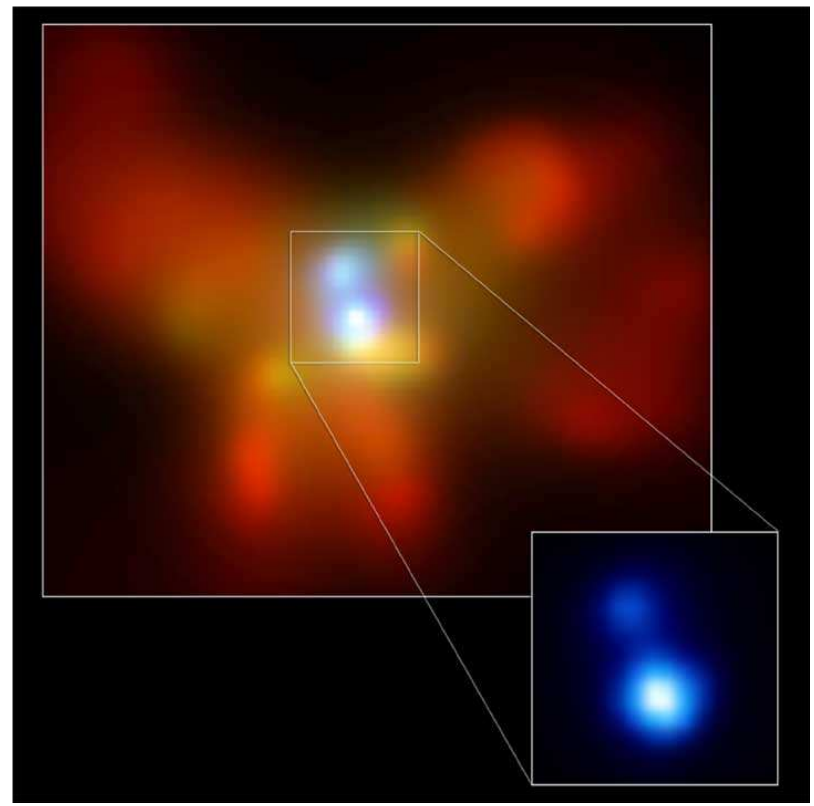
[Hartle, Phys. Rev. D **9**, 2749 (1974)]

So what? — Binary BH and gravitational waves

The tidal heating of black holes has observational consequences in the context of gravitational-wave astronomy.

Consider a system of two black holes in orbital motion. . .





[http://www.obspm.fr]

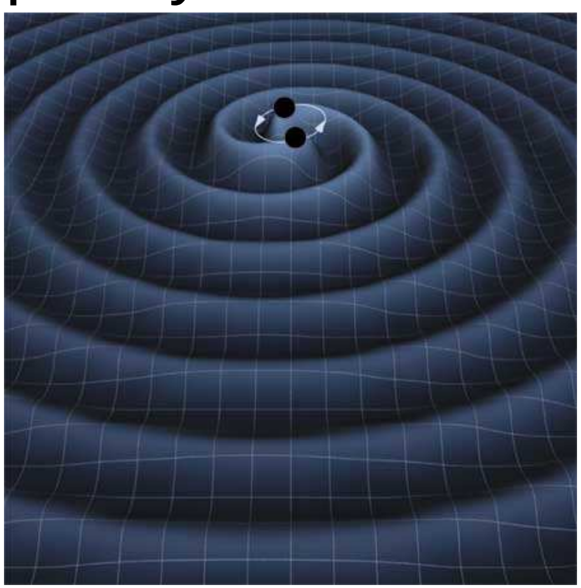
[http://www.msfc.nasa.gov]

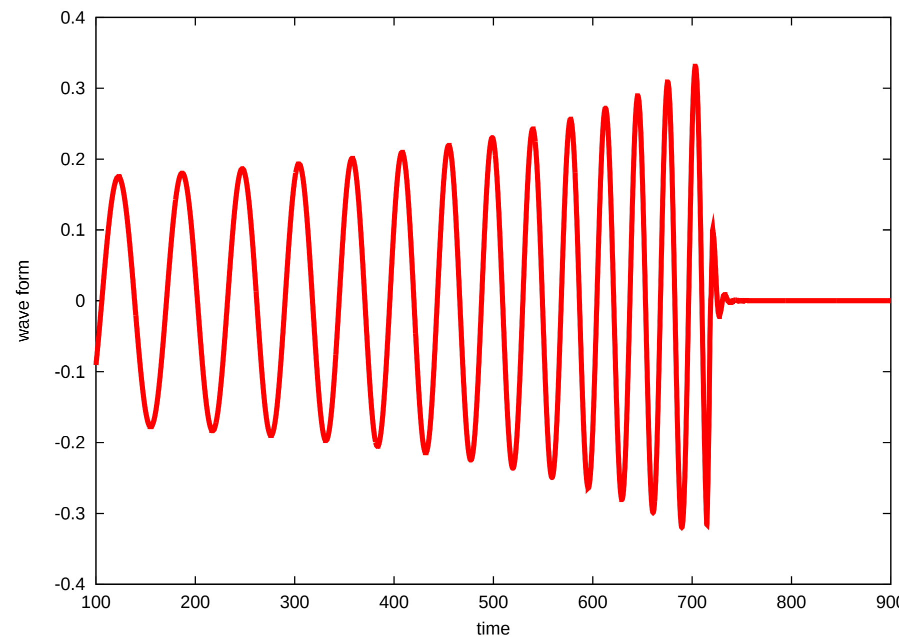
So what? — Binary BH and gravitational waves

Such a system emits gravitational waves, and the radiated energy comes at the expense of the orbital energy.

The radiation reaction of the orbital motion is driven by $dE_{\rm orb}/dt < 0$: the orbit shrinks and the orbital frequency increases.

This is reflected in the gravitational-wave signal: its frequency also increases.





[http://lisa.jpl.nasa.gov/gallery/]

So what? — Binary BH and gravitational waves

But tidal heating also is involved in the energy balance,

$$\frac{dE_{\rm orb}}{dt} = -\frac{dE_{\rm rad}}{dt} - \frac{đQ_1}{dt} - \frac{đQ_2}{dt}$$

The effect must be taken into account.

For holes in circular orbits,

$$\frac{dE_{\rm rad}}{dt} \simeq \frac{32}{5}\frac{c^5}{G}\frac{{M_1}^2{M_2}^2}{(M_1+M_2)^4}(v/c)^{10}$$

$$\frac{đQ_1}{dt} \simeq \frac{32}{5}\frac{c^5}{G}\frac{{M_1}^6{M_2}^2}{(M_1+M_2)^8}(v/c)^{18}$$

The effect is small for slow orbits, but it is significant for holes moving rapidly in a strong gravitational field.

Conclusion

- Tidal heating is responsible for Io's volcanic activity.
- The tidal heating of a black hole is governed by closely analogous physics.
- The effect has observational consequences for gravitational-wave astronomy.