# Wave propagation in curved spacetime

## *Hearing the shape of a black hole and the geometry of the Universe*

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# Outline

- Gravitational waves
- Gravitational-wave detectors
- Coalescing compact binaries
- Wave propagation near a black hole
- Shape of the event horizon
- Geometry of the Universe
- Conclusion

# Gravitational waves

Gravitational waves are ripples in the fabric of spacetime that propagate (at the speed of light) away from their point of origin.

They are produced by accelerated massive bodies; large masses and high speeds are required.

The passage of a gravitational wave induces tiny oscillations in the relative separation between two (or more) test masses.

These can be measured by laser interferometry.

Most gravitational-wave signals of astrophysical origin are coherent; measuring them is analogous to hearing.

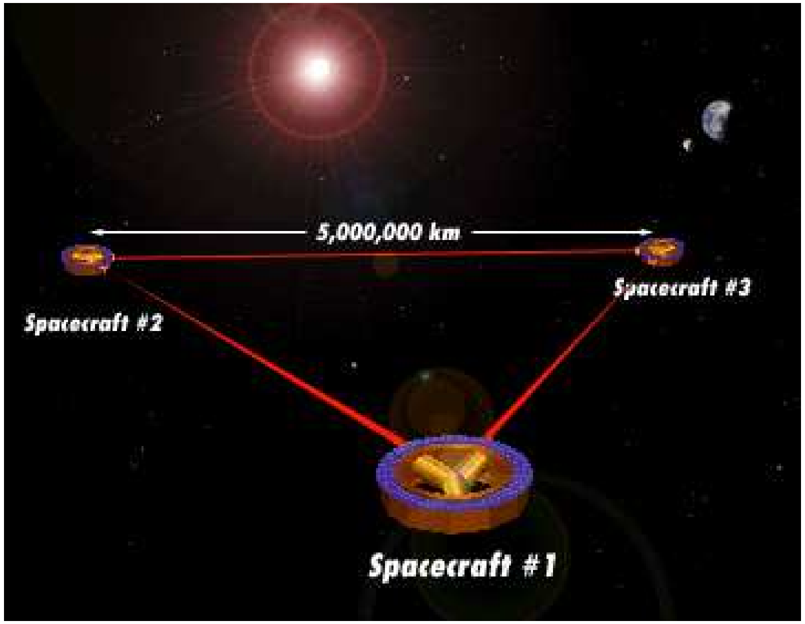
# Gravitational-wave detectors

Laser Interferometer Gravitational-wave Observatory (LIGO)

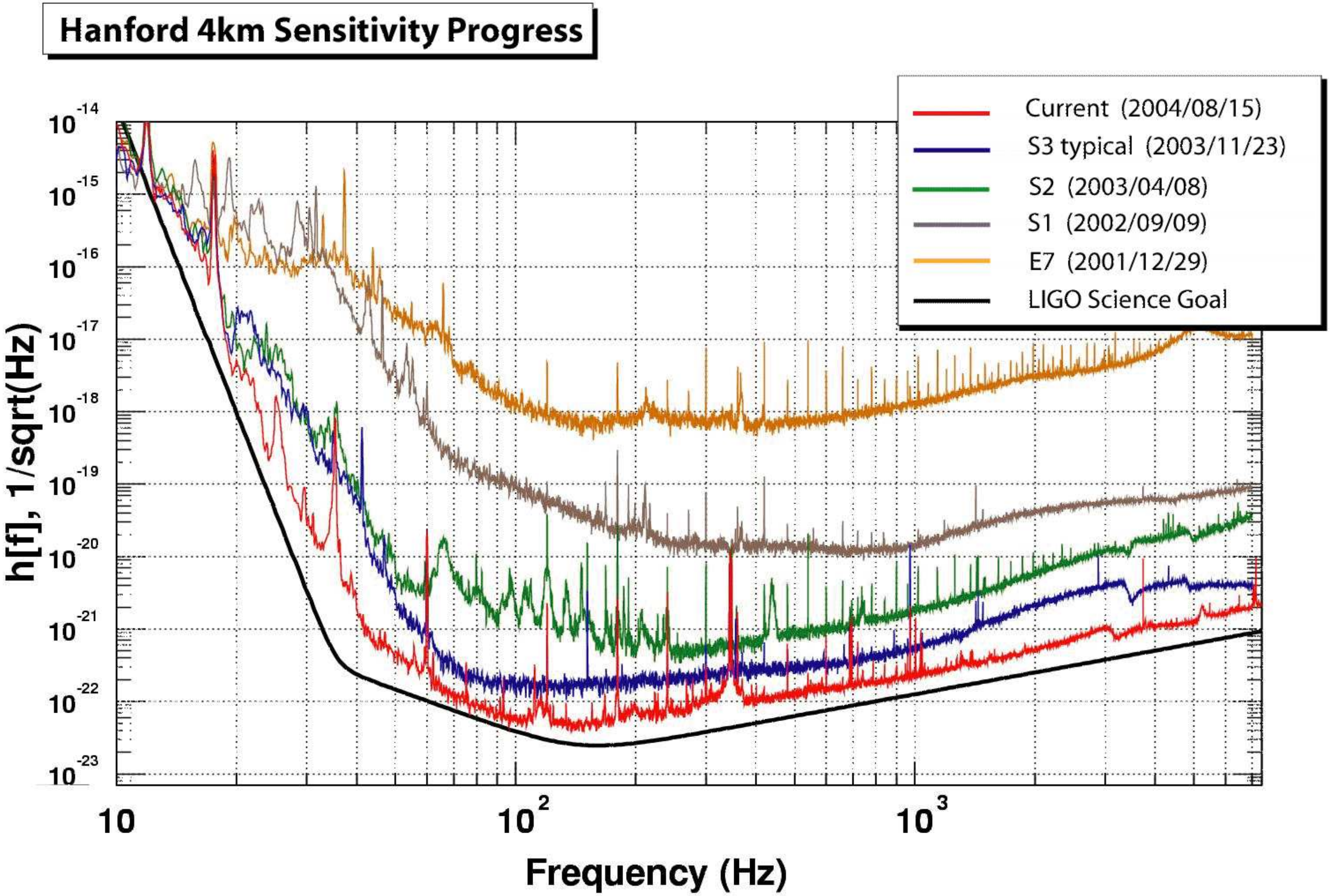
$10$ Hz $< f < 1\,000$ Hz

Laser Interferometer Space Antenna (LISA)

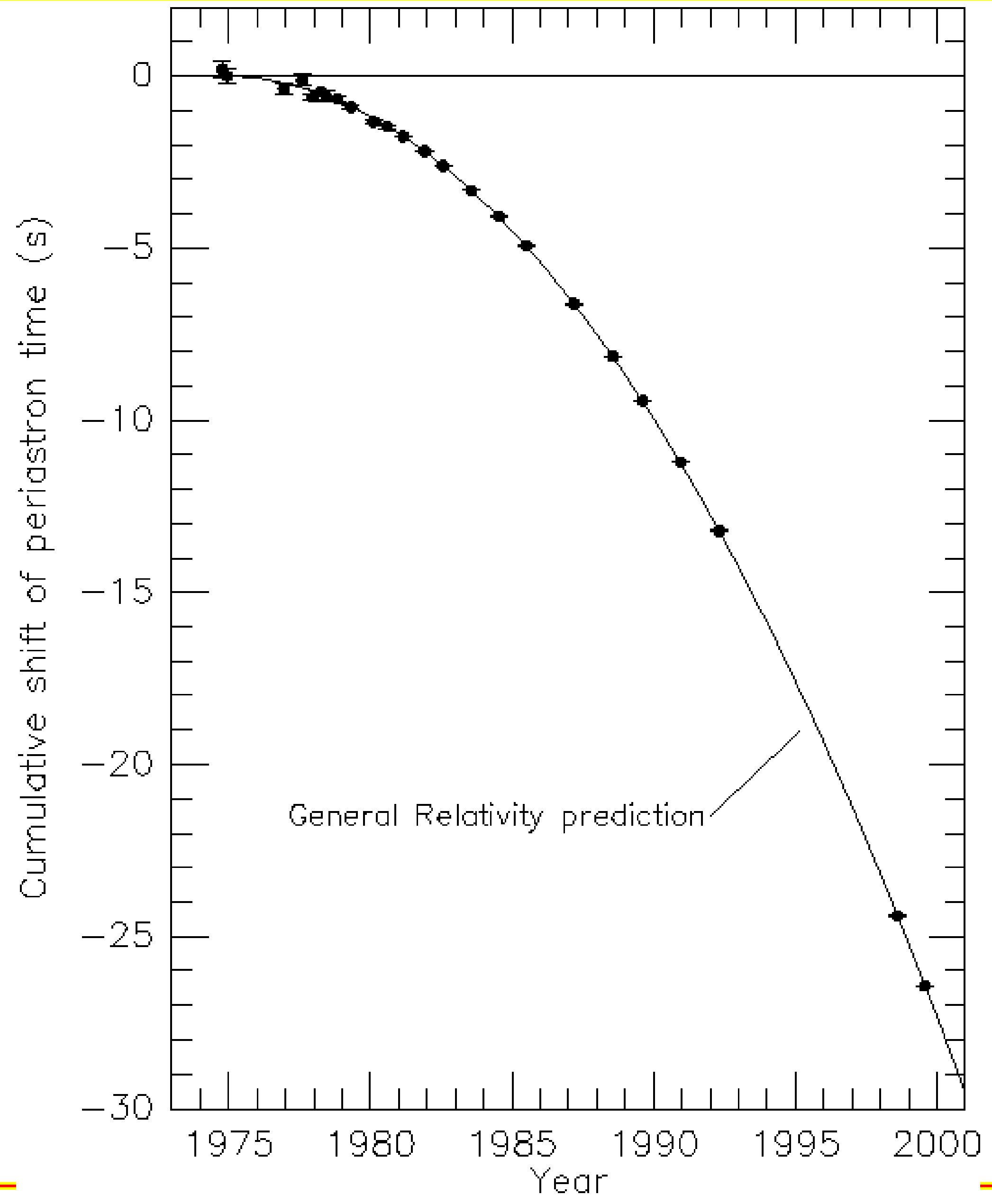
$0.1$ mHz $< f < 100$ mHz



# Gravitational-wave detectors



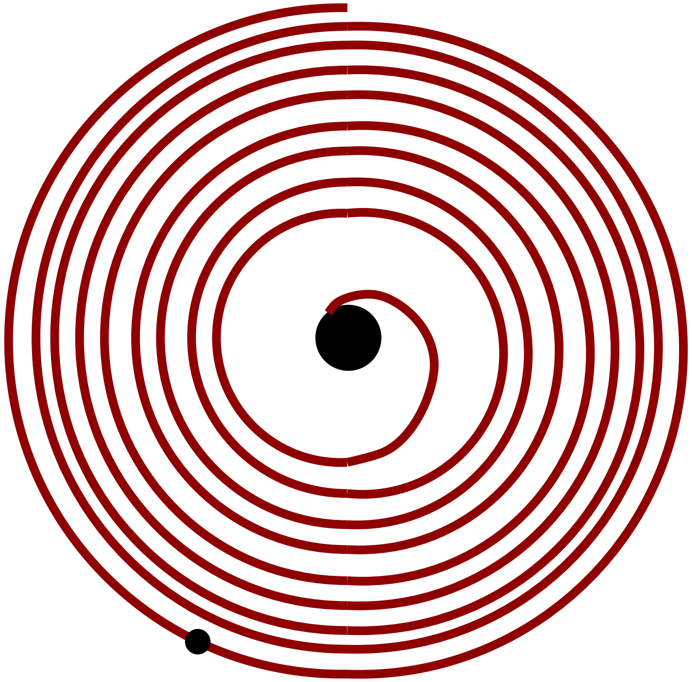
# Gravitational-wave detectors



Radiation reaction
in the Hulse-Taylor
binary pulsar
PSR 1913+16

# Coalescing compact binaries

A promising source of gravitational waves for both LIGO and LISA is the late stages of orbital evolution of a compact binary system.



(For LIGO imagine a 1 $M_\odot$ neutron star on a tight orbit around a 50 $M_\odot$ black hole; for LISA imagine a 50 $M_\odot$ black hole on a tight orbit around a $10^6$ $M_\odot$ black hole.)

# Coalescing compact binaries

A signal coming from such a system displays three distinct phases:

- A long **inspiral phase**, during which the orbit tightens under radiation reaction.
  This ends when the orbit becomes unstable and the compact object plunges into the central black hole.

The inspiral phase reveals details of the binary system’s composition (masses, spins, etc.) and state of motion.

# Coalescing compact binaries

- A short **shake-down phase**, during which the central black hole, tidally distorted by the infalling object, shakes off the distortion and returns to a stationary state.

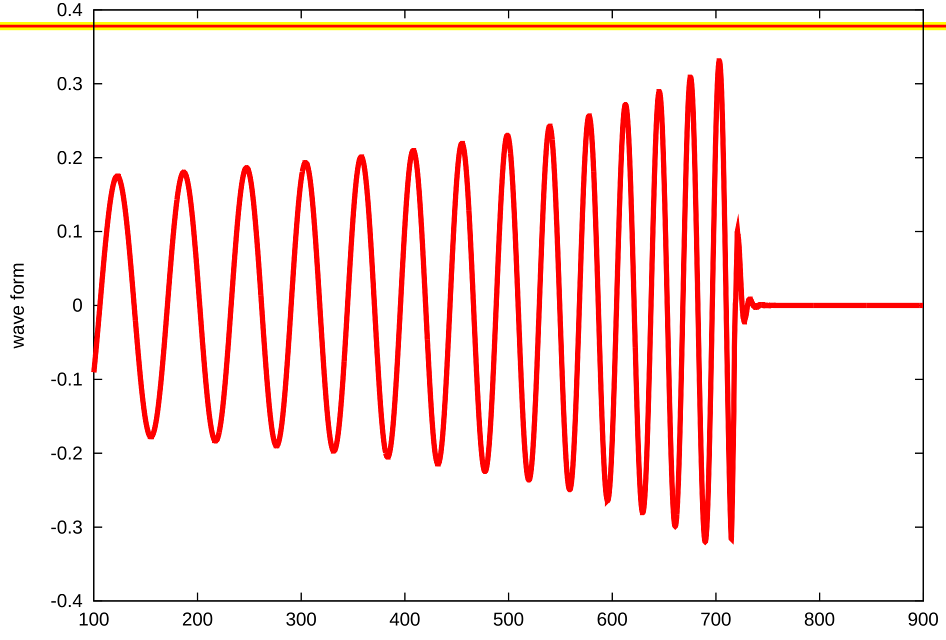
The shake-down phase reveals details of the event horizon.

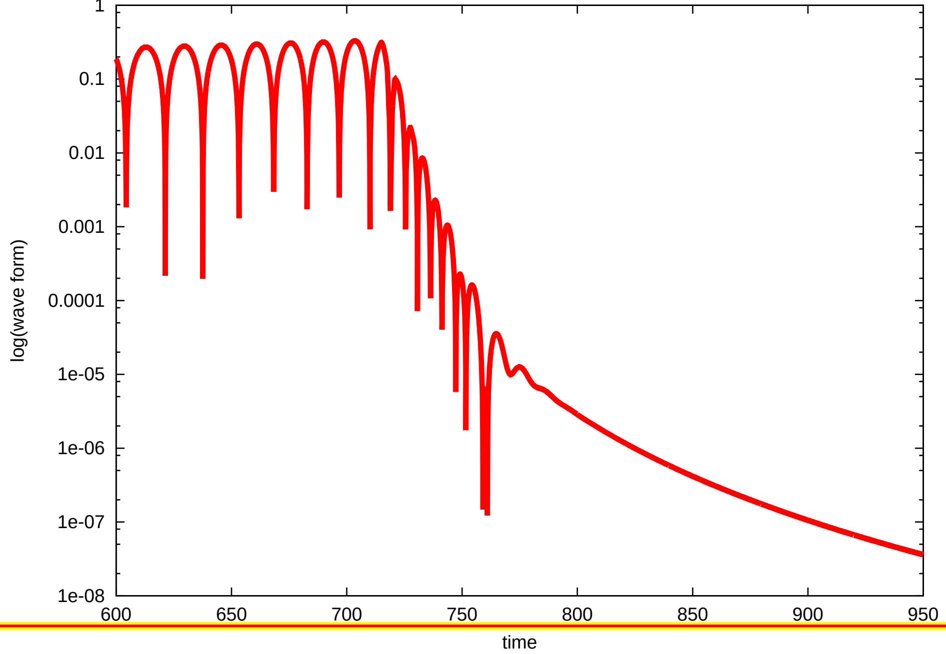
# Coalescing compact binaries

- A never ending **quiet-down** phase, during which a portion of the signal that propagated out to large distances returns back to the detector.

The quiet-down phase reveals details of the structure of spacetime at large distances — the geometry of the Universe.

# Coalescing compact binaries





# Wave propagation near a black hole

Wave propagation in the spacetime of a nonrotating black hole can be analyzed via a decomposition into spherical harmonics.

The reduced wave equation describes waves propagating in the radial direction, subjected to a curvature-generated potential barrier.

# Wave propagation near a black hole

The equation is simplified by a stretching of the radial coordinate,

$$\begin{aligned} r^* &= r + R\ln(r/R - 1) \\ R &= \frac{2GM}{c^2} \equiv \text{Schwarzschild radius} \end{aligned}$$

which pushes the event horizon to $r^* = -\infty$.

# Wave propagation near a black hole

The reduced wave equation takes the form of

$$\left[-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial r^{*\,2}}-V(r)\right]\psi(t,r)=0$$

where $\psi(t,r)$ is the wave function and

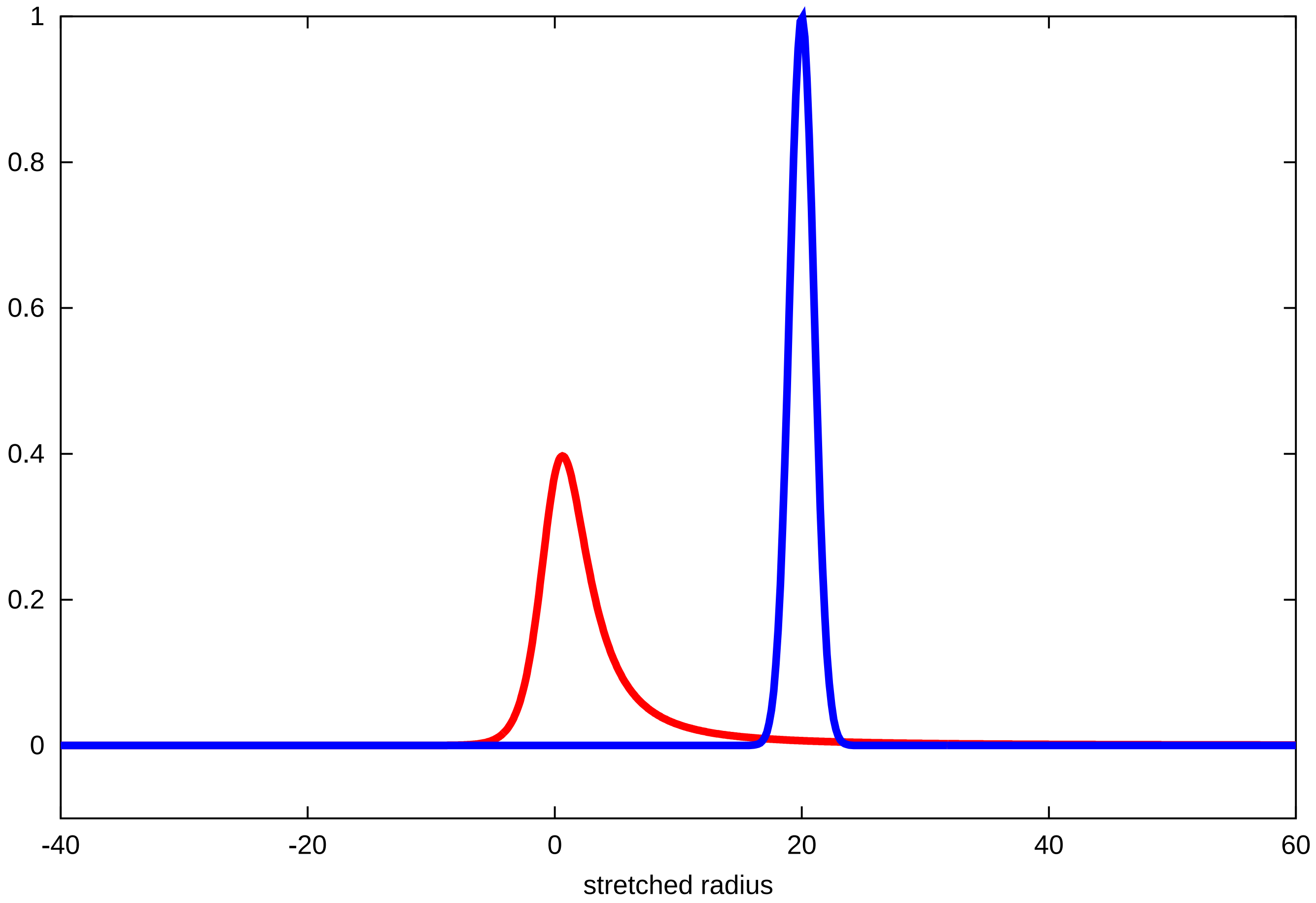
$$V(r)=\left(1-\frac{R}{r}\right)\left[\frac{\ell(\ell+1)}{r^2}-\frac{3R}{r^3}\right]$$

is the potential barrier, which is peaked near $r^*=0$, or $r=\frac{3}{2}R$.

# Wave propagation near a black hole

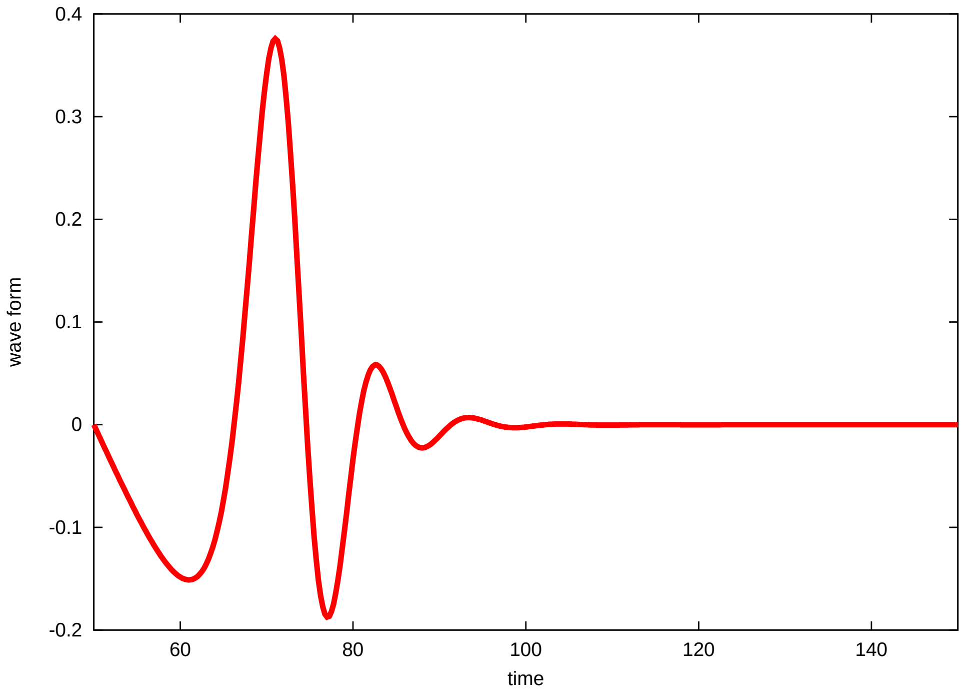
To see how waves propagate near a black hole, we will examine the evolution of an **imploding wave**. [movie]

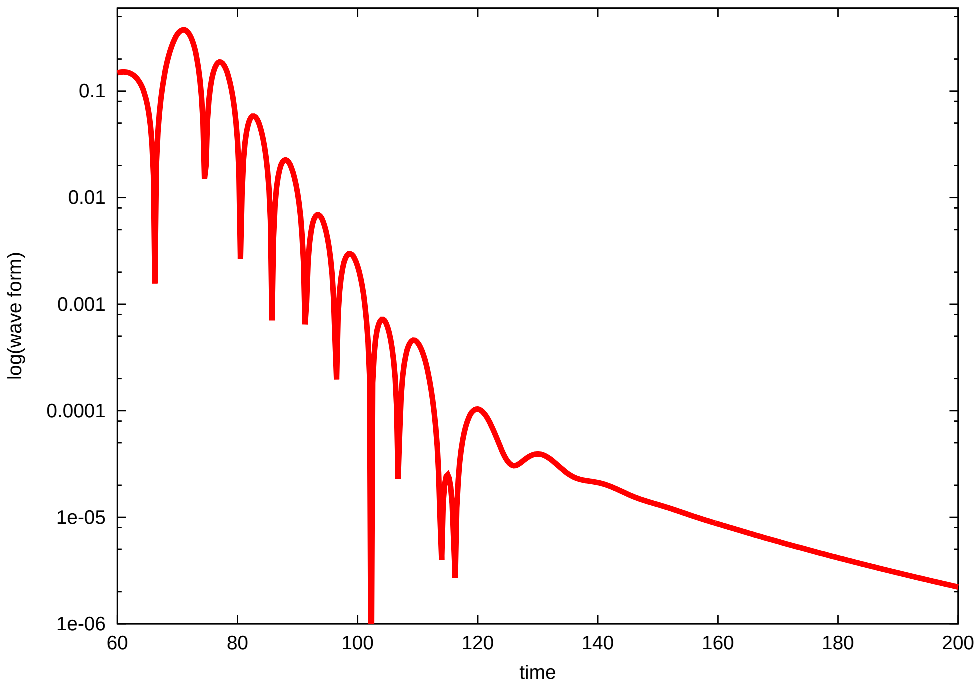
initial wave profile



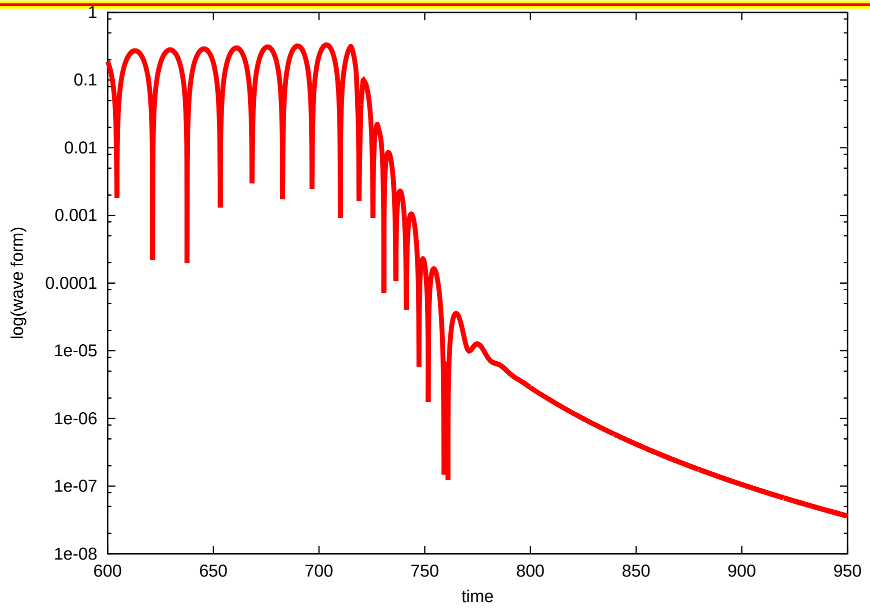
# Wave propagation near a black hole

Measured signal as a function of time:

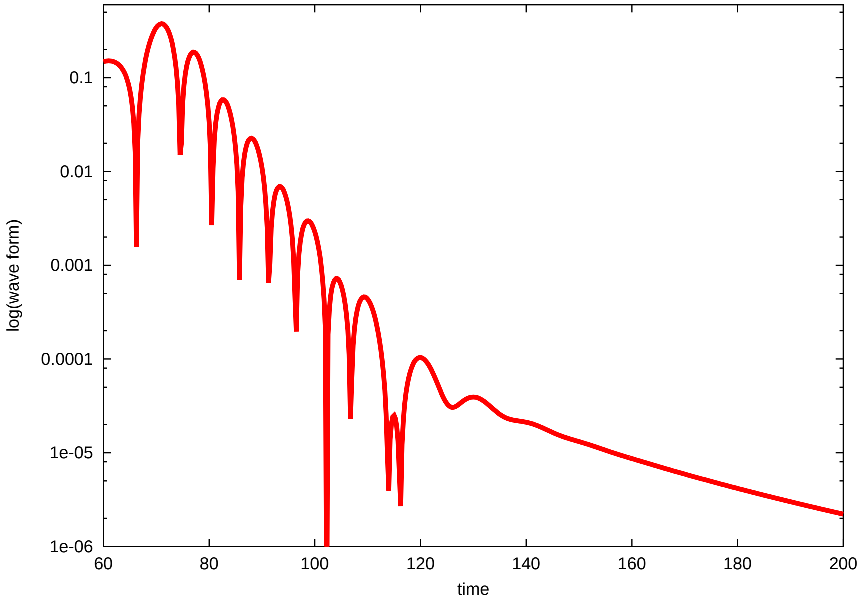




# Wave propagation near a black hole



signal from binary system



signal from imploding wave

# Shape of the event horizon

The **shake-down phase** of the signal is characterized by exponentially-decaying oscillations.

This is a wave-propagation effect produced by the interaction of the wave with the high concentration of curvature near the event horizon.

The frequency of the oscillations, and the decay constant of their amplitude, bring detailed information concerning the shape of the event horizon.

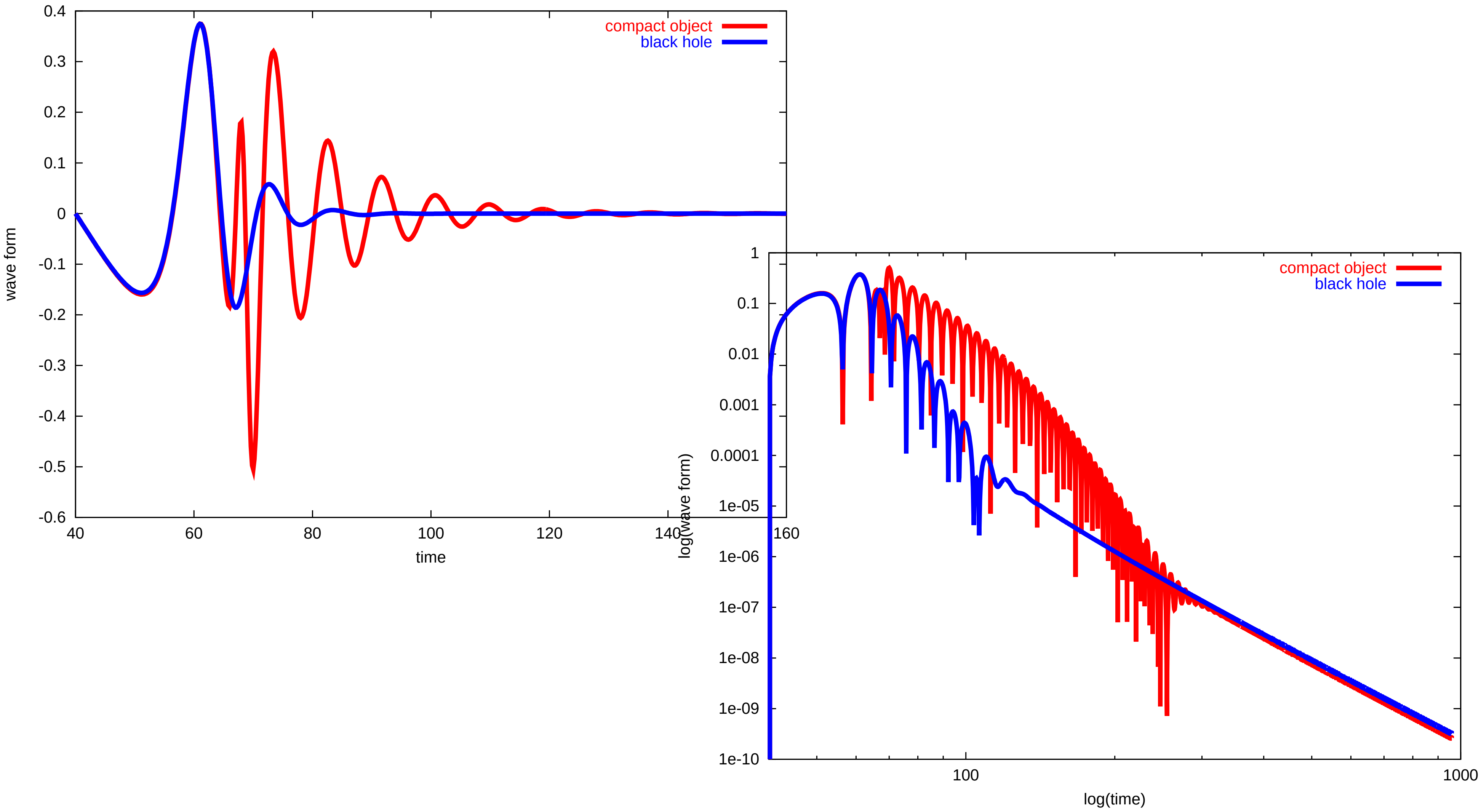
In fact, these features depend sensitively on the presence of an event horizon; a compact object without an event horizon would give rise to a very different wave form.

# Shape of the event horizon

**Gravitational-wave measurements provide a means to directly detect the presence of an event horizon and thus prove the existence of black holes.**

Imagine waves propagating near a compact object whose radius is 10% larger than its Schwarzschild radius. . .

# Shape of the event horizon



# Geometry of the Universe

The **quiet-down phase** of the signal begins when the damped oscillations stop.

This is characterized by an inverse power-law decay of the wave's amplitude.

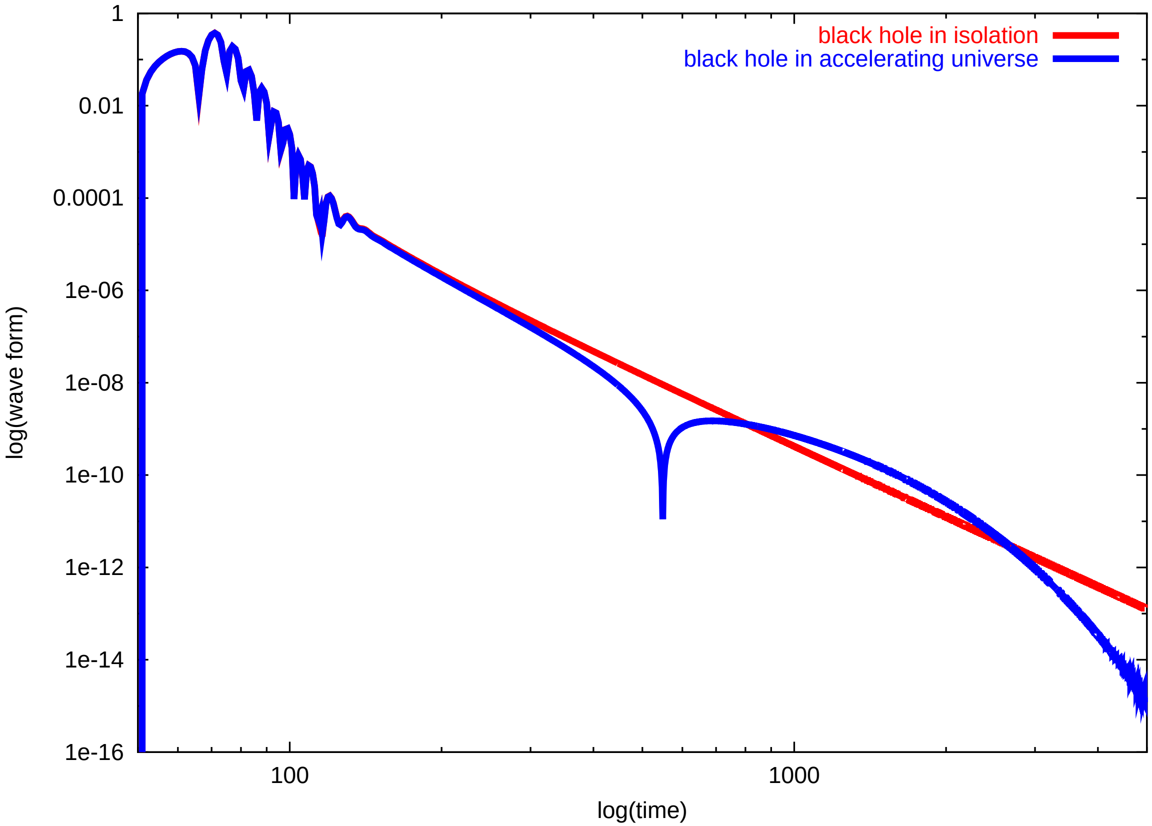
This is another wave-propagation effect, produced by the residual curvature that is still present at very large distances from the hole.

(The wave propagates out to a very large distance, undergoes some backscattering, and comes back toward the detector.)

This decay brings information about the spacetime's asymptotic structure — the geometry of the Universe.

# Geometry of the Universe

Imagine waves propagating around a black hole immersed in an accelerating Universe…



(But the effect is very small and differences occur only after a Hubble time.)

# Conclusion

The measurement of gravitational waves will reveal how waves propagate in the strongly curved spacetime of a black hole.

Theoretical predictions will be compared with measurements, and many strong-field aspects of general relativity will thus be amenable to tests.

The existence of event horizons will directly be probed (something that conventional astronomy cannot do well).

And in principle (but this is $\pi$ in the sky), wave-propagation effects allow the determination of the large-scale structure of the Universe.