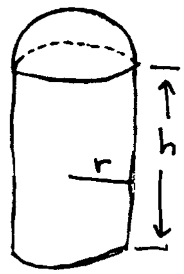
Problem 10-29

(a)



volume of cylinder $= \pi r^2 h$

volume of hemisphere $= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$

$\therefore$ total volume $= \pi r^2 h + \frac{2}{3}\pi r^3$

area of cylinder = circumference × height

$= 2\pi r h$

area of hemisphere $= \frac{1}{2}(4\pi r^2) = 2\pi r^2$

$\therefore$ total area $= 2\pi r h + 2\pi r^2$

(b) Let $\frac{h}{r} = c$ (constant) $\therefore$ $h = cr$

From (a), volume $V = \pi r^2 h + \frac{2}{3}\pi r^3$

$$= \pi r^2 (cr) + \frac{2}{3}\pi r^3$$

$$= \left(\pi c + \frac{2}{3}\pi\right) r^3$$

$$= D r^3, \text{ where } D = \pi c + \frac{2}{3}\pi = \text{const.}$$

Initial volume $V_1 = D r_1^3$

New volume $\underbrace{V_2} = 2 V_1$

$$\therefore \quad D r_2^3 = 2 D r_1^3 \Rightarrow r_2 = \sqrt[3]{2}\, r_1$$

Then, $h_2 = c r_2 = c \sqrt[3]{2}\, r_1 = \sqrt[3]{2}\,(c r_1) = \sqrt[3]{2}\, h_1$

New area $A_2 = 2\pi r_2 h_2 + 2\pi r_2^2$

$$= 2\pi (\sqrt[3]{2}\, r_1)(\sqrt[3]{2}\, h_1) + 2\pi (\sqrt[3]{2}\, r_1)^2$$

$$= (\sqrt[3]{2})^2 \left[2\pi r_1 h_1 + 2\pi r_1^2\right]$$

$$= 2^{2/3} A_1$$

(c) ... continued ...

Problem 10-29 (c)

Volume $V = \pi r^2 h + \frac{2}{3}\pi r^3$

Initial $V_1 = \pi r_1^2 h_1 + \frac{2}{3}\pi r_1^3$

Radius remains constant at r_1

$\therefore$ New $V_2 = \pi r_1^2 h_2 + \frac{2}{3}\pi r_2^3$

But $V_2 = 2V_1$

$$\therefore \pi r_1^2 h_2 + \frac{2}{3}\pi r_2^3 = 2(\pi r_1^2 h_1 + \frac{2}{3}\pi r_1^3)$$

$$= 2\pi r_1^2 h_1 + \frac{4}{3}\pi r_1^3$$

$$\therefore \pi r_1^2 h_2 = 2\pi r_1^2 h_1 + \frac{2}{3}\pi r_1^3$$

$$\therefore h_2 = 2h_1 + \frac{2}{3} r_1$$

New area $A_2 = 2\pi r_1 h_2 + 2\pi r_1^2$

$$= 2\pi r_1 \left(2h_1 + \frac{2}{3} r_1\right) + 2\pi r_1^2$$

$$= 4\pi r_1 h_1 + \frac{4\pi}{3} r_1^2 + 2\pi r_1^2$$

$$= 4\pi r_1 h_1 + \frac{10\pi}{3} r_1^2$$

Rewrite this in terms of $2A_1 = 2(2\pi r_1 h_1 + 2\pi r_1^2)$

$$= 4\pi r_1 h_1 + 4\pi r_1^2$$

$$\therefore A_2 = \underbrace{4\pi r_1 h_1 + 4\pi r_1^2}_{2A_1} \underbrace{- 4\pi r_1^2 + \frac{10\pi}{3} r_1^2}_{-\frac{2\pi}{3} r_1^2}$$

$$\therefore A_2 = 2A_1 - \frac{2\pi}{3} r_1^2$$