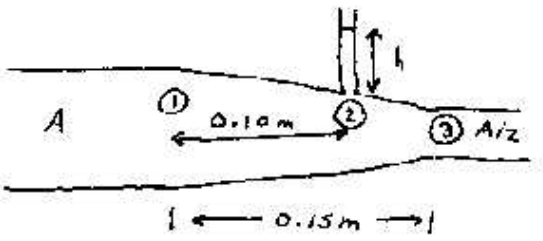
Problem 12-6



Height h will be determined by pressure $P_2 = \rho g h$ [1]

Find P_2 using Bernoulli's Eq'n -

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

But $y_1 = y_2$ (centre of tube has constant elevation)

$$\therefore P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2 \quad [2]$$

$P_1, \rho,$ & v_1 are known. $\therefore$ to find P_2, need v_2 1st.

Use Eq'n. of continuity: $A_1 v_1 = A_2 v_2$ [3]

$\therefore$ must determine A_2 (to find v_2)

From right to left, tube area increases from $A/2$ to A in distance of 0.15m (from point 3 to point 1).

$\therefore$ rate of change of area with distance is:

$$\frac{A - A/2}{0.15\text{m}} = \frac{A/2}{0.15\text{ m}} = \frac{A}{0.30}\ \text{m}^{-1}$$

$\therefore$ increase in area in distance of 0.05m (from point 3 to point 2 is $\left(\frac{A}{0.30}\text{m}^{-1}\right)(0.05\text{m}) = \frac{A}{6}$

$$\therefore A_2 = \frac{A}{2} + \frac{A}{6} = \frac{2A}{3}$$

(A_2 at point 2; $\frac{A}{2}$ at point 3)

$\therefore$ in [3], $A v_1 = \frac{2A}{3} v_2 \therefore v_2 = \frac{3}{2} v_1 = 0.15\text{m/s}$

$\therefore$ in [2], $50 + \frac{1}{2}(1000)(0.10)^2 = P_2 + \frac{1}{2}(1000)(0.15)^2$

$$\Rightarrow P_2 = 43.8\ \text{Pa}$$

$\therefore$ in [1], $43.8 = (1000)(9.8)h$

$$\therefore h = 0.0045\ \text{m}$$