

Exercise 9-26

Initial grav. P.E. of mass m = final K.E. of m + final KE of cylinder

$$\therefore mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \quad [1]$$

For a cylinder, $I = \frac{1}{2} M R^2$ [2] (Table 9-1)

speed v of mass m = speed v of cable

= speed v of rim of cylinder

$$\therefore v = \omega R \quad \text{or} \quad \omega = \frac{v}{R} \quad [3]$$

Subst. [2] & [3] into [1]:

$$\therefore mgh = \frac{1}{2} m v^2 + \frac{1}{2} \left(\frac{1}{2} M R^2 \right) \frac{v^2}{R^2}$$

$$\therefore mgh = \frac{1}{2} v^2 \left(m + \frac{1}{2} M \right)$$

Divide both sides by $m \Rightarrow gh = \frac{1}{2} v^2 \left(1 + \frac{M}{2m} \right)$

$$\therefore v = \sqrt{\frac{2gh}{1 + \frac{M}{2m}}}$$

$$\text{From [3], } \omega = \frac{v}{R} = \frac{1}{R} \sqrt{\frac{2gh}{1 + \frac{M}{2m}}}$$