August 7, 2007

THE FORCE BETWEEN CURRENT-CARRYING CONDUCTORS

OBJECTIVE: To investigate the nature of the force between two parallel wires which are carrying current, and to measure the value of the permeability of free space. Doing this experiment will give you an appreciation of the remarkable experimental skill of Ampère, who was the first to measure this force accurately, in 1820.

REFERENCE: Duffin, Chapter 7
Griffiths, Chapter 5

APPARATUS: The current balance
A laser
Scale
Variac
6.3 volt 10 amp transformer
Ammeter 10 amp max.

THEORY:

The force on a current element $I_2 d\vec{l}_2$ in a magnetic field $\vec{B}$ is given by

$$d\vec{F}_{12} = I_2 d\vec{l}_2 \times \vec{B} \qquad (1)$$

The force on a current element $I_2 d\vec{l}_2$ due to an element $I_1 d\vec{l}_1$ can be obtained by considering the magnetic field $\vec{B}_1$ created at the second element by the first:

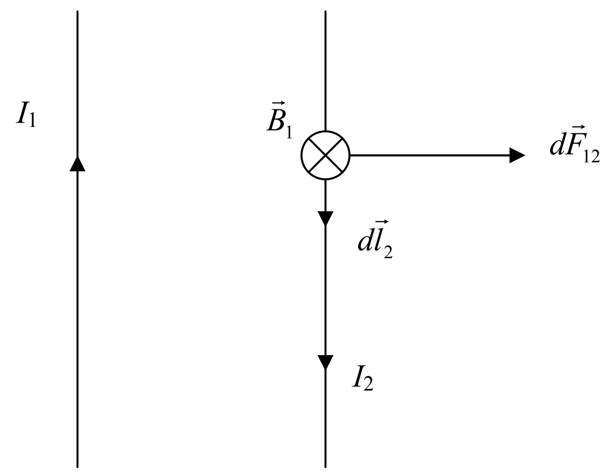
$$\vec{B}_1 = \frac{\mu_o}{4\pi r^2} I_1 d\vec{l}_1 \times \hat{r}_{12} \qquad (2)$$

where $\hat{r}_{12}$ is the unit vector pointing from $d\vec{l}_1$ to $d\vec{l}_2$. Combining (1) and (2) gives

$$d\vec{F}_{12} = \frac{\mu_o}{4\pi r^2} I_1 I_2 d\vec{l}_2 \times (d\vec{l}_1 \times \hat{r}_{12}) \qquad (3)$$

which is the force on $d\vec{l}_2$ due to the current in $d\vec{l}_1$.

Consider the case of two infinitely long wires separated by a distance d, carrying currents I_1 and I_2, as shown in the figure below.



The magnetic field due to current I_1 anywhere along the wire carrying current I_2 is given by:

$$\left|\vec{B}_1\right| = \frac{\mu_o I_1}{2\pi d}$$

which can be derived via Ampère's law. The right hand rule tells us that $\vec{B}_1$ is directed into the page along the wire carrying current I_2. If we choose an element $d\vec{l}_2$ anywhere along the wire carrying I_2, the magnetic field $\vec{B}_1$ and $d\vec{l}_2$ are perpendicular, as shown. Therefore, equation (1) becomes:

$$\left|d\vec{F}_{12}\right| = I_2 dl_2 B_1$$

or

$$\left|d\vec{F}_{12}\right| = I_2 dl_2 \frac{\mu_o I_1}{2\pi d}$$

and the direction of $d\vec{F}_{12}$, determined by the right hand rule, is to the right in the figure, *i.e.* away from the wire carrying I_1.

Integrating along the length of the wire, the total force on length L is given by:

$$\left|\vec{F}_{12}\right| = \frac{\mu_o I_1 I_2}{2\pi d} L$$

or

$$\left|\vec{F}_{12}\right| = \frac{\mu_o L}{2\pi d} I^2 \qquad (4)$$

if $I_1 = I_2$.

The force is repulsive (as shown) if I_1 is in the opposite direction to I_2, and attractive otherwise.

In this experiment, you will investigate experimentally the way in which F varies with I for two parallel conductors, and derive a value for μ_o.

The Current Balance

The apparatus you will use is called a "Current Balance". It consists of a fixed conductor and a parallel conductor which is on a counter-balanced rocker so that the spacing d may be altered. The moveable conductor carries a small pan upon which small weights may be placed. The rocker also carries a mirror so that by means of an optical lever the separation d may be measured or (as is done in the diagram) be maintained constant.

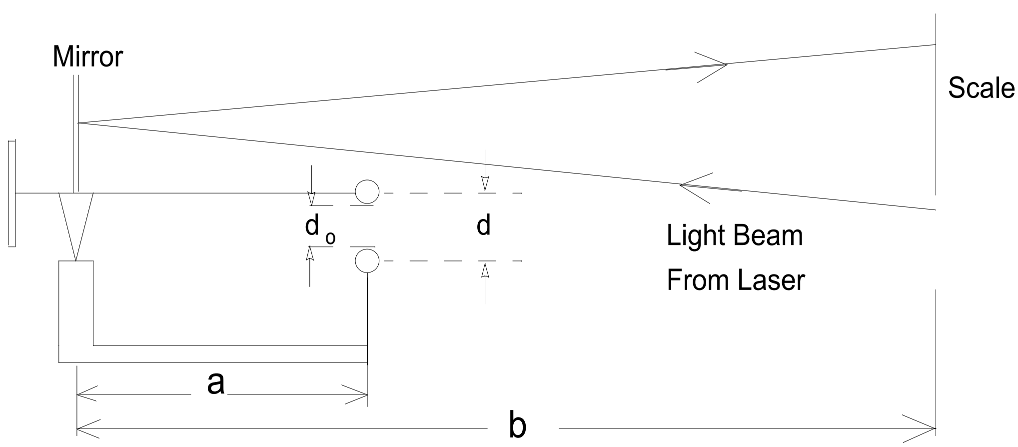


Figure 2

Figure 2 shows this optical setup and defines several important distances. The diameter of the conductors is 2r, and d is the distance between their centres.

EXPERIMENT:

1. Set up the circuit for the apparatus as shown in Figure 3.

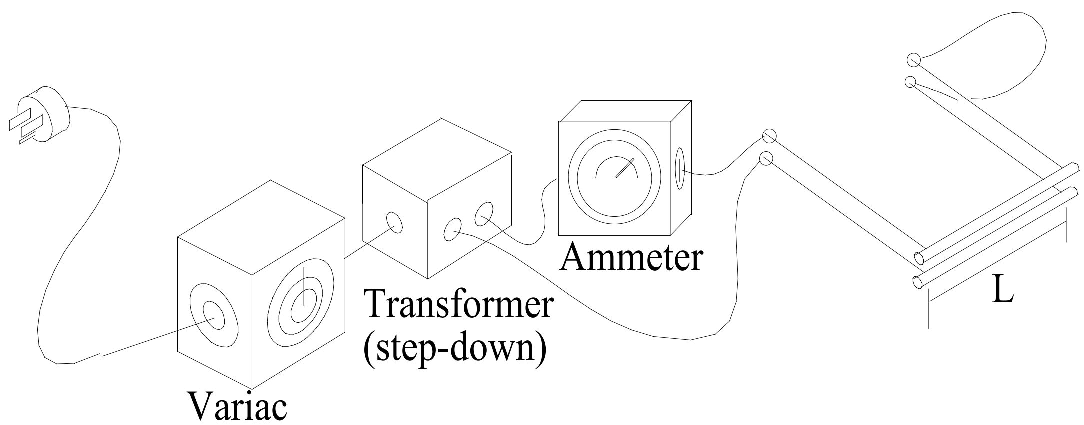


Figure 3

2. The two conductors must be straight and parallel. By placing a coin on the pan, you can ensure that the conductors touch and thus check their parallelism. Sufficient adjustments are provided to ensure this.

3. Referring to parameters indicated in Figures 2 and 3, measure and record a, b, and L. Measure and record the diameter of the wire and the diameter of the laser beam on the ruler. Record all relevant uncertainties.

4. Next you must balance the lever arm so that the conductors are about <u>2 mm</u> apart. You can determine this distance d_o by recording the <u>scale readings</u> when the conductors touch and when they are in equilibrium. Their difference <u>D</u> gives d_o by means of the relation

$$d_o = \frac{Da}{2b}$$

Derive this relation.

5. The main experiment consists of adding small weights (10 mg up to 100 mg; uncertainty ± 0.5 mg) and determining the current required to maintain a fixed separation d_o. Generate at least 10 data points for analysis.
   <u>**NOTE:**</u> **Do not go over 10 amp.**

6. Plot F vs I^2 and comment on the result. Obtain the slope of this graph and deduce μ_o.

ANALYSIS:

A complete analysis of the experimental errors is required. Some of the more significant sources of error that should be assessed and accounted for are as follows:

The derivation of the expression $d_o = Da/2b$ requires that the ruler and the laser are aligned in the same plane. However, it is very difficult to do this and ensure that the distance b is same for both the ruler and laser. Typically, the laser is located adjacent to the ruler and therefore, the incident and reflected path length are not in the same plane. Try to estimate the uncertainty this would contribute to the measurement of b. In addition, it is usually difficult to avoid a slight angle in the measuring tape when measuring b. This will introduce additional uncertainty to the measured b and should also be accounted for.

The validity of the infinite wire approximation should also be discussed. Since the separation of the wires is quite small compared with the length of the wires, approximating the length of the wires to be infinite should be valid. However, a nonuniform spacing between the wires will contribute a considerable amount of uncertainty to the validity of these approximations and to the accurate measurement of the separation of the wires. It may also be instructive to repeat the measurement of the diameter of the wires at different points along the wires length. Use the differences in these values to estimate the uncertainty in the separation of the wires.

The diameter of the light beam spot on the ruler is quite large and this will contribute additional uncertainty to the measurement of D.

The length L of the wires should only be measured from the portion of the wires that are conducting (i.e., not from the extreme ends). This will introduce additional uncertainty to the measured L.

When estimating the effect of these uncertainties, you may find it difficult to quantify them exactly. It may be useful to do some additional measurements that are not included in the procedure. This list is, by no means, meant to be comprehensive and any additional relevant sources of error you can think of should also be discussed.

EXPECTATIONS FOR THE FORCE BETWEEN CURRENTS LAB

- Derive $d_o = \frac{D\,a}{2\,b}$; include a diagram
- State measurements of *D, a, b, L*, 2*r*, with uncertainties.
- Tabulate the masses and currents. Justify your errors on the masses and currents.
- Calculate I^2, propagate errors, then tabulate with the force.
- Perform weighted linear regression to find the line of best fit for *F* vs. I^2.
- State slope and intercept with uncertainty.
- Plot the data with the fitted line you get from the regression.
- Calculate d_o, calculate μ_o, both with errors.
- Comment on the validity of the small angle approximation.
- In your discussion/ conclusion:

-compare to theory
-comment on the intercept
-comment on the systematic errors