Energy dissipation in tapping-mode atomic force microscopy

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A method is presented to measure the energy dissipated by the tip–sample interaction in tapping-mode atomic force microscopy (AFM). The results show that if the amplitude of the cantilever is held constant, the sine of the phase angle of the driven vibration is then proportional to changes in the tip–sample energy dissipation. This means that images of the cantilever phase in tapping-mode AFM are closely related to maps of dissipation. The maximum dissipation observed for a 4 N/m cantilever with an initial amplitude of 25 nm tapping on a hard substrate at 74 kHz is about 0.3 pW. © 1998 American Institute of Physics.

Students of classical dynamics are taught that there are often two ways to study a mechanical system: through forces or energy. Considering the name of the field, it is not surprising that in atomic force microscopy (AFM) the dominant viewpoint has always been the Newtonian one. There has been much interest in imaging material properties by measuring the phase of a tapping cantilever relative to the drive, and recent modeling of tapping-mode AFM has all centered on writing down a force-based equation of motion for the cantilever that includes nonlinear terms to account for the tip–sample interaction and then solving the differential equation (usually numerically). See Refs. 6–11 for some recent work in this area. In this letter, we show that by taking advantage of the observed steady-state motion of the cantilever and calculating the energy inputs and outputs, the phase of the oscillating cantilever can be directly related to the energy loss in the tip–sample junction, and a numerical value for the power being lost can be extracted.

Our interest in the energy viewpoint came about because of a few surprises in some computer modeling of tapping-mode AFM done by one of the authors and co-workers. The model qualitatively reproduced several of the features seen in experimental amplitude and phase versus distance curves even though the only energy loss in the model corresponded to air damping of the body of the cantilever. Also, like the experimental data (Fig. 1), the steady-state solution of the tapping cantilever was nearly sinusoidal even though a (conservative) highly nonlinear force dependence was introduced to model the tip–sample interaction. This led us to analyze the role energy dissipation plays in tapping-mode AFM and to the realization that the energy inputs and outputs can be easily accounted for as long as the motion stays nearly sinusoidal.

In equilibrium the average rate at which energy is fed into the cantilever must equal the average rate at which energy is dissipated by the cantilever and the tip. It is convenient to break the dissipated power into two pieces, so that 

\[ P_{\text{in}} = P_0 + P_{\text{tip}}. \]

The first piece of the dissipated power \( P_0 \) can be thought of as “background” dissipation that is present but that we are not interested in directly. In most cases, \( P_0 \) covers power dissipated by the body of the cantilever (e.g., air damping) and is well modeled by simple viscous damping, while \( P_{\text{tip}} \) includes sources of dissipation localized to a small volume including the tip and sample.

We first calculate the input power, \( P_{\text{in}} \) term for a cantilever with spring constant \( k \) whose base position \( z_d(t) \) is driven sinusoidally with amplitude \( A_d \) and a frequency \( \omega \). Assuming a sinusoidal steady-state response, the deflection from equilibrium of the end of the cantilever, \( z(t) \), can be written as \( A \cos(\omega t + \varphi) \) where \( A \) is the amplitude of the cantilever and \( \varphi \) is the phase of the cantilever relative to the driver. The instantaneous power delivered by the driver is the force on the driver times the velocity of the driver:

\[ P_{\text{in}} = F_d \dot{z}_d = k [z(t) - z_d(t)] \dot{z}_d. \]

Integrating over a complete cycle, the average power yields

\[ \overline{P_{\text{in}}} = \frac{1}{2} k A_d A \omega \sin \varphi. \]  

This contains the familiar result that the maximum power is delivered to an oscillator when the response is 90° out of phase with the drive.

Now let us address the power leaving the cantilever. Assuming that the background dissipation \( P_0 \) is well modeled by viscous damping of the cantilever body, \( F_{\text{damping}} = b \dot{z} \), a similar analysis yields the average background power

\[ \overline{P_0} = \frac{1}{2} b A^2 \omega^2. \]  

We can now solve for the power dissipated by the tip, since experimentally, it is easiest to measure \( b \) by measuring the cantilever spring constant \( k \), and then measuring the quality factor \( Q_{\text{cant}} \) and natural resonant frequency \( \omega_0 \) from the shape and frequency of the free-cantilever resonance, we use \( Q_{\text{cant}} = k/b \omega_0 \) to get

\[ \overline{P_{\text{tip}}} = \frac{1}{2} k A^2 \omega \left[ \frac{Q_{\text{cant}} A_d \sin \varphi}{A} - \omega \right]. \]

Note that the reason that \( Q_{\text{cant}} \) and \( \omega_0 \) appear in Eq. (3) is to express the viscous damping coefficient \( b \) in terms of experimentally accessible quantities. Equation (3) does not imply that the resonant frequency or \( Q \) of the interacting cantilever remains the same; it only assumes that the viscous damping coefficient describing the damping of the body of the lever remains unchanged.

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In this case, further simplification occurs because we can ask
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energy carried in the second cantilever resonance at 475 kHz; over 50% of the fundamental peak
including higher harmonics not shown in the figure, so the motion stays sinusoidal to a good approximation. There is very little energy carried in the second cantilever resonance at 475 kHz; over 50% of the power there is due to thermal excitation of the mode.

Most commonly, the drive frequency is chosen to be \( \omega_0 \).
In this case, further simplification occurs because we can define the free amplitude of the cantilever as
\( A_0 = Q_{\text{cant}} A_{\text{d}} \)
(this is true even for \( Q_{\text{cant}} < 1 \), yielding
\[
\frac{P_{\text{tip}}^0}{2} = \frac{kA^2 \omega_0}{Q_{\text{cant}}} \left( \frac{A_0}{A} \right) \sin \varphi - 1. \tag{4}
\]

The computer model provided a specific case in which expression (4) could be verified. Since the modeled tip–sample interaction was conservative (i.e., \( P_{\text{tip}} = 0 \), the expression in square brackets in Eq. (4) was expected to be zero, or equivalently \( \varphi = \arcsin(A/A_0) \). This relationship was well obeyed in the modeling data.

One of the most important consequences of Eq. (4) is that, if the tip loses no energy, then the amplitude and the phase are not independent. Experimentally, phase imaging is performed with the amplitude held constant by a feedback loop, so it is only when the tip–sample interaction losses vary that phase contrast will be observed. Some recent modeling by our group\(^{15}\) and by Tamayo and Garcia\(^{16}\) agrees with this. The one exception in which phase contrast is not due to dissipation arises because it is the \( \sin \varphi \) rather than \( \varphi \) itself that appears in Eq. (4). Since sine is a symmetric function about 90°, phase changes symmetric about 90° are allowed even if there are no losses in the tip–sample interaction. Such symmetric jumps from attractive (>90°) to repulsive phases (<90°) are observed in the modeling data when conservative tip–sample interactions are used.\(^5\)\(^^{16}\) In standard phase imaging, the phase angle is plotted, so phase contrast observed where the phase jumps from attractive phases to repulsive phases is not due to dissipation, but is instead due to competition between the attractive and repulsive forces.

Equation (4) tells us how we can interpret phase images taken on resonance at constant amplitude. The \( \sin \varphi \) is simply proportional to the power being lost by the tip plus a constant. As long as the phase stays on one side of 90° (the microscope is always operating in the attractive or the repulsive regime), then any changes in the phase image are directly due to changes in the energy being dissipated in the tip–sample junction. In fact, if a few parameters are measured, a phase image can be directly converted into a quantitative image of dissipation.

We used expression (4) to evaluate the energy dissipated by a 225 \( \mu \text{m} \) silicon cantilever\(^\text{17}\) tapping on a silicon wafer in ambient conditions. The \( k \), \( Q_{\text{cant}} \), and \( \omega_0 \) (4.0 N/m, 130, and \( 2\pi \cdot 73881 \text{ Hz} \)) of the “free” cantilever were evaluated by measuring and fitting the thermal noise.\(^{18,19}\) Ideally, \( k \) is measured far enough from the sample so that there is no interaction and \( Q_{\text{cant}} \) is measured close enough to include squeeze film damping effects.\(^20\) Then, amplitude and phase versus distance curves were recorded (Fig. 1) and Eq. (4) was used to calculate the power being dissipated.

There are a couple of features of Fig. 2(c) that are immediately interesting. One is the maximum power dissipated of about 0.3 pW. This corresponds to about 25 eV per tap of the cantilever. Although this is sufficient energy to break several covalent bonds, if the contact area is just a few nanometers across, the energy per atom becomes a fraction of an electron volt, explaining the ability of tapping-mode AFM to image many samples nondestructively. The other is the fact that in the repulsive phase region the energy dissipation is independent of sample position (i.e., amplitude). This is consistent with the modeling, which showed that the peak force on the sample was fairly constant after the transition to repulsive phases.\(^6\) The 0.45 pW peak in the retract curve occurs just before the tip breaks free of the capillary force. The maximum noise in the dissipation data due to amplitude and phase noise is 20 fW (1 kHz bandwidth); however, systematic errors can be larger. A precise determination of \( \omega_0 \) is important because this is used to determine the offset in the phase data. For high \( Q \) cantilevers, errors of several Hz can translate into absolute phase errors of a few degrees. For the data presented here, the error due to this effect is estimated to be 50 fW.

The assumptions in the derivations are, that the steady-state motion of the cantilever is sinusoidal, and the damping on the body of the lever is described by a simple viscous damping coefficient that remains unchanged when the tip begins interacting with the sample. Also, only the first mode
of the oscillating cantilever is necessary to describe the steady-state motion. Note that no assumptions have been made concerning the nature of the tip–sample interaction, although only certain tip–sample interactions will yield nearly sinusoidal steady-state motion. The measured motion was very close to sinusoidal for both silicon and polyethylene samples with cantilevers ranging from 4 to 100 N/m in stiffness. The fact that the steady-state oscillation is nearly sinusoidal does not imply the system is linear. In fact, observation of the transients shows highly nonlinear behavior. Some insight into why the system can evolve into a nearly sinusoidal state is provided by the study of grazing impact oscillators.\textsuperscript{21,22}

Although the data presented here were for a cantilever tapping at a surface, the analysis presented works in any case where the cantilever oscillation remains close to sinusoidal. For noncontact microscopies such as electric force microscopy (EFM) or magnetic force microscopy (MFM), it can be more convenient to feedback on the frequency to keep the lever on resonance. In that case, the phase is held constant and changes in $P_{\text{tip}}$ appear in the cantilever amplitude. Grueter \textit{et al.} have recently measured dissipation in MFM by such a method.\textsuperscript{23} In some earlier work, Denk and Pohl\textsuperscript{24} measured electrical dissipation by measuring the width of the cantilever resonance.

One important thing to note is the method presented here will only provide the total energy lost in the tip–sample interaction, not how or where it is lost. Figuring out specific places the energy is going will require modeling and creative experimentation. A similar problem arises in AFM force curves. The force on the tip can be the total of several forces acting simultaneously on the tip (e.g., capillary forces, van der Waals forces, magnetic forces), but we only get one measured quantity: the total force on the tip. Yet, by looking at how this total force depends on distance, or what happens when we add salt to the solvent, we can start to guess the separate contributions to this force. The same thing should be possible with energy dissipation.

In conclusion, we have shown a means for measuring the energy dissipated by the tip of an oscillating AFM cantilever. The ability to measure the energy dissipated in nanometer-sized contacts should prove to be useful in many respects, but the most important practical result of the work is that tapping-mode AFM phase images can now be interpreted in terms of energy dissipation.

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{The amplitude (a), phase (b), and powder dissipation (c), of the resonating cantilever measured as the sample was approached (solid lines) and retracted (dotted lines). The lettered arrows in (a) point to amplitudes at which the corresponding spectra in Fig. 1 were taken. The regions labeled in (b) show where the tip is experiencing overall attractive or repulsive forces.}
\end{figure}

13. Unpublished data. The largest harmonic distortion was, typically, –40 to –60 dB.
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