Absorption and Transmission of light and the Beer-Lambert Law

Lecture 21

www.physics.uoguelph.ca/~pgarrett/Teaching.html

Review of L-20

- Ring wave functions (circumference ℓ)

$$\left.\begin{aligned}\Psi_{s,n}(x) &= \sqrt{\frac{2}{\ell}}\sin\left(\frac{2\pi nx}{\ell}\right)\\ \Psi_{c,n}(x) &= \sqrt{\frac{2}{\ell}}\cos\left(\frac{2\pi nx}{\ell}\right)\end{aligned}\right\} n=1,2,3,\ldots$$

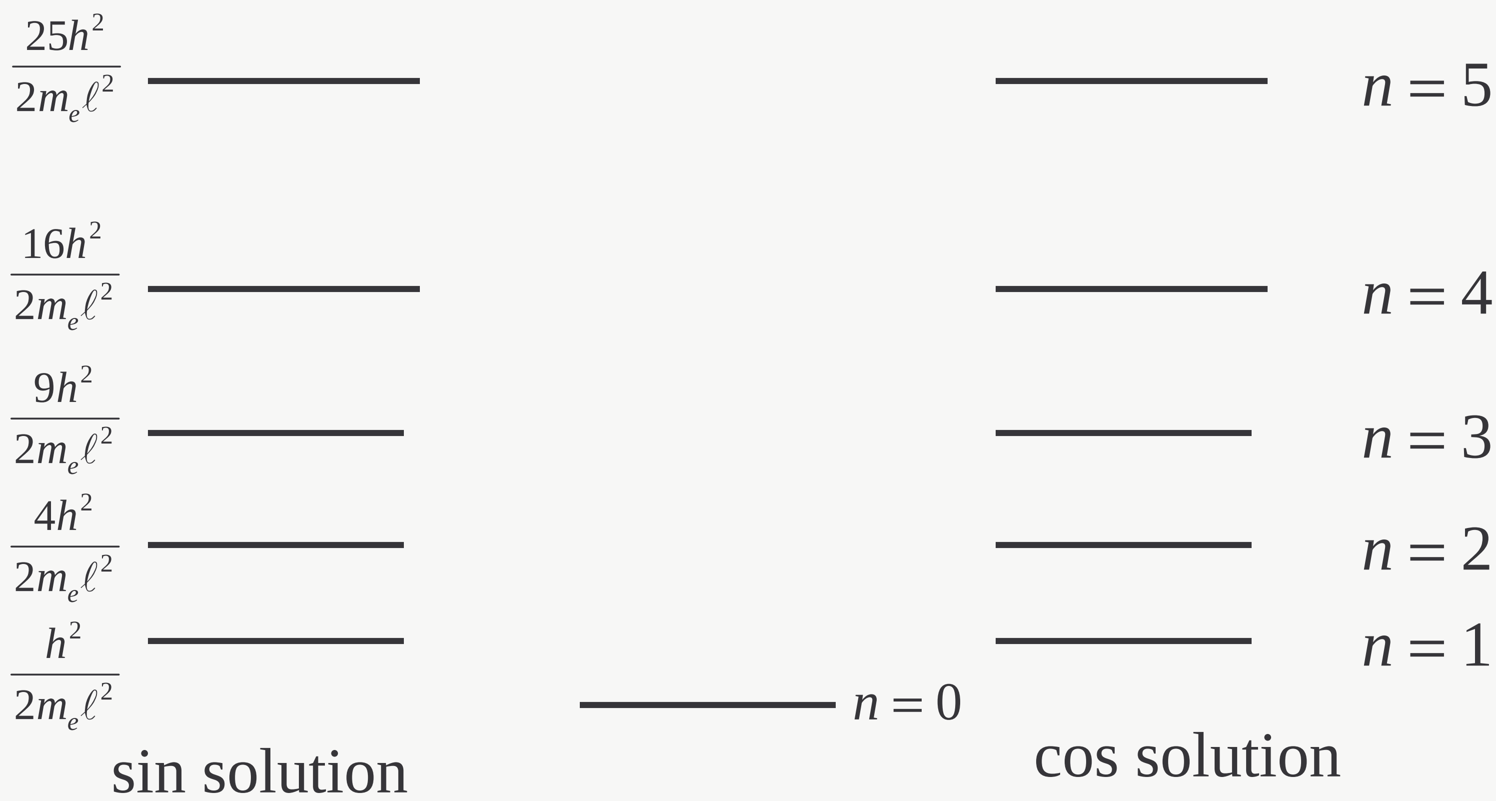
$$\Psi_{c,0} = \sqrt{\frac{1}{\ell}} \quad (n=0)$$

- Energy levels

$$E_n = \frac{n^2 h^2}{2m_e \ell^2}$$

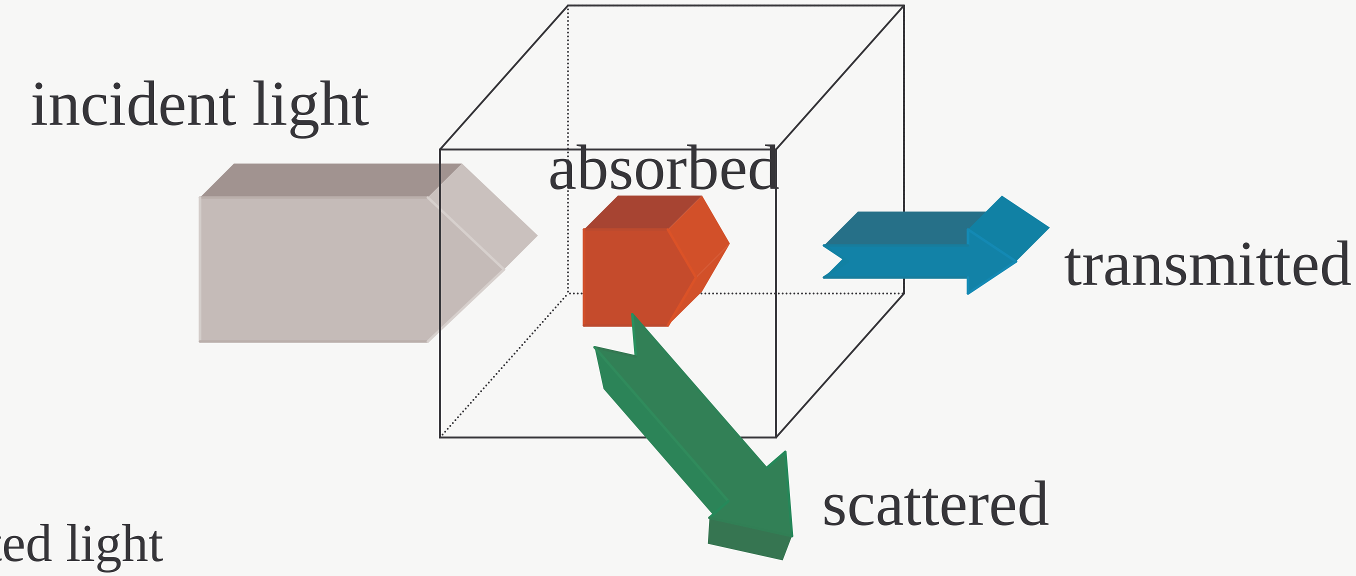
Review of L-20

- Energy levels in a ring molecule



Absorption and Transmission

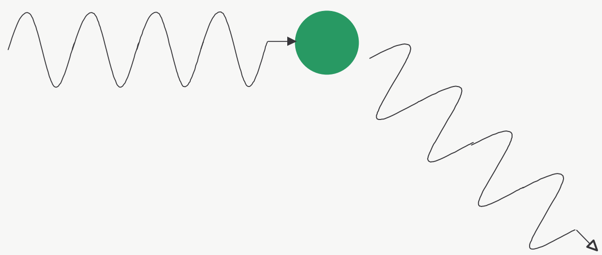
- Consider a beam of light on a material
  - It can be scattered, absorbed, or transmitted

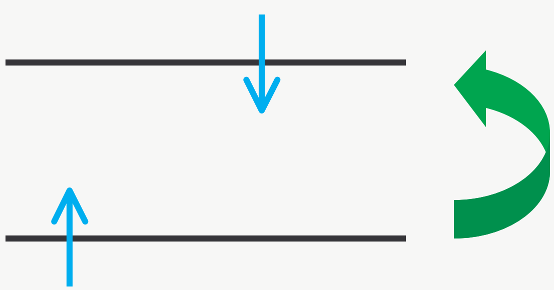


- Transmitted light
  - Light emerges propagating in the same direction as the incident light
- Absorbed light
  - Energy from light is absorbed in the volume of the material
- Scattered light
  - Light emerges in a different direction from the incident light

Absorption and Transmission

- Absorption and scattering take place at the molecular and atomic level
  – For energy from light to be absorbed, it must match available energy states in the atoms or molecules, or it can scatter from the molecule, atom, or electrons, … (like billiard balls)





ex. Promotion of an e^{-} to an excited state

Cross sections

- Light incident upon a material *may or may not* be removed from the incident beam
- Probability that the light is removed from the incident beam is related to the *cross section* σ
  - Cross section σ is like an effective area of the atom or molecule
  - σ can be larger or smaller than the geometric cross section
  - σ takes into account the absorption and scattering of light

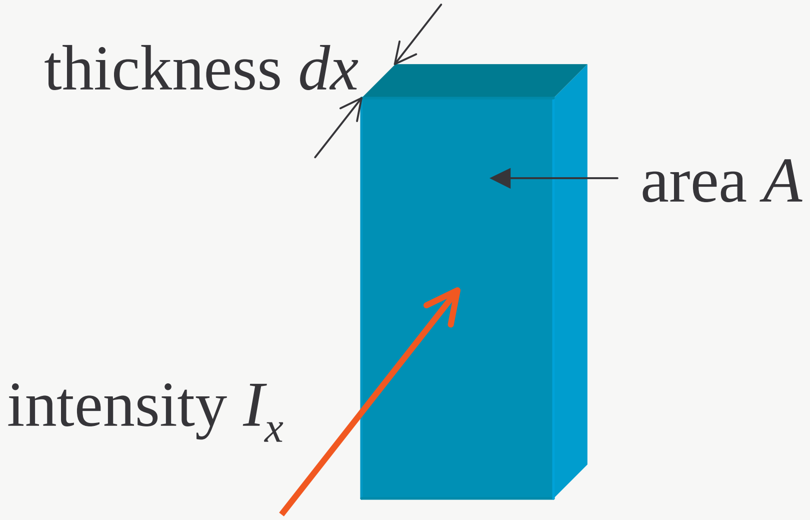
B2 Stealth bomber of the US has a radar cross section smaller than that of hummingbird

Beer-Lambert Law

- Consider light incident on a material with area A and thickness dx and concentration of molecules C (i.e. # / cm^3)
- Number of molecules illuminated by light of incident intensity I_x is $CAdx$
- Total "effective" area that the molecules present is $\sigma CAdx$
- Probability of light being absorbed or scattered out of the beam in thickness dx is

$$-\frac{dI_x}{I_x} = \frac{\sigma CA}{A} dx$$

- where dI_x is the change in intensity across dx



Beer-Lambert Law

- We have $-\frac{dI_x}{I_x} = \frac{\sigma CA}{A} dx$
- Now integrate both sides

$$\int_{I_0}^{I} \frac{dI_x}{I_x} = -\int_0^x \sigma C dx$$

$$\ln(I) - \ln(I_0) = \ln\left(\frac{I}{I_0}\right) = -\sigma Cx$$

$$I = I_0 e^{-\sigma Cx} = I_0 e^{-\mu x}$$

- The coefficient $\mu = \sigma C$ is the *linear attenuation coefficient*
  - If we ignore scattering, we can equate the linear attenuation coefficient with the *linear absorption coefficient*

Beer-Lambert Law

- The Beer-Lambert Law

$$I = I_0 e^{-\sigma C x} = I_0 e^{-\mu x}$$

- The intensity of light decreases exponentially with depth in the material
- Linear attenuation coefficient μ usually expressed in units of cm^{-1}
- μ is a function of wavelength ⇒ μ=μ(λ)
  - So Beer-Lambert Law is also a function of λ, i.e.

$$I(\lambda) = I_0(\lambda)\ e^{-\mu(\lambda)\, x}$$

Transmittance

- Transmittance is defined as $T = \dfrac{I(\lambda)}{I_0(\lambda)}$
- A quantity called *absorbance A* is defined as

$$A = \log\left(\frac{I_0(\lambda)}{I(\lambda)}\right) = \log\left(\frac{I_0(\lambda)}{I_0(\lambda)e^{-\mu x}}\right) = \log\left(e^{\mu x}\right)$$

$$A = \mu x \log(e) = 0.4343\mu x$$

- This definition includes both absorption and scattering
- A further definition is the *extinction coefficient* ε

$$\varepsilon = 0.4343\sigma$$

so

$$A = \varepsilon C x$$

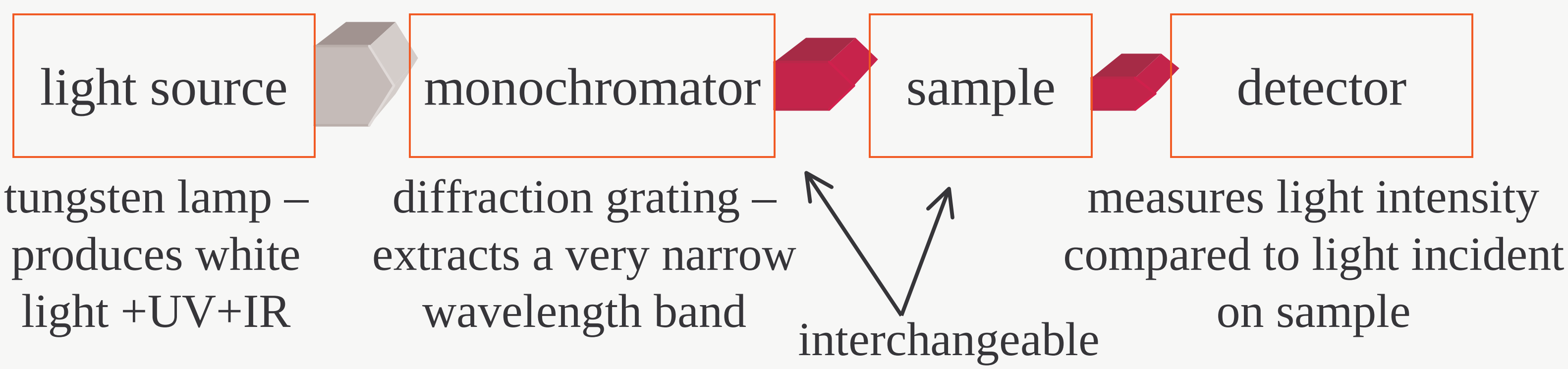
Absorbance and the extinction coefficient

- Absorbance is useful since it can be summed for layers of different materials, each with their own *x*, *C*, σ, etc.

$$A_{tot} = A_1 + A_2 + A_3 + \ldots$$

$$A_{tot} = \varepsilon_1 C_1 x_1 + \varepsilon_2 C_2 x_2 + \ldots$$

- A specialized device to measure the intensity of light as a function of wavelength is the *spectrophotometer*
  - Come in various varieties – absorption, fluorescence, …



Absorption spectrum

- Absorption spectra for chlorophyll