Radioactive decay and half life

Lecture 25

www.physics.uoguelph.ca/~pgarrett/teaching.html
Review of L-24 – Fluorescence

- Property of some atoms or molecules to absorb light at a particular wavelength and then emit light at a longer wavelength (lower frequency) than the incident light.
Review of L-24 – Phosphorescence

- Emission of delayed longer wavelength (than fluorescence) radiation

\[
\begin{align*}
\text{Singlet states} & : n = 0, 1, 2, 3, \ldots \\
\text{Triplet states} & : n = 0, 1, 2, 3, \ldots \\
\end{align*}
\]

- Electron in excited state
- Visible or UV radiation
- Electron left in ground state
- Long lifetime – must wait until electron spin flips back again
- Longer wavelength radiation
At centre of atoms, nucleus containing $Z$ protons and $N$ neutrons

- Total number of nucleons (either protons or neutrons) $A$
  \[ A = Z + N \]
- $A$ is also known as the atomic mass number because for a mole of atoms, the molar mass is
  \[ M_M \approx A \text{ g/mol} \]
- ex. For a mole of atomic oxygen with $A=16$
  \[ M_M = 15.99491 \text{ g/mol} \approx 16 \text{ g/mol} \]
- ex. For a mole of molecular oxygen $O_2$ with $A=16$
  \[ M_M = 31.98982 \text{ g/mol} \approx 32 \text{ g/mol} \]
Atomic nucleus

- Mass of proton
  - \( M_p = 1.673 \times 10^{-27} \text{ kg} \)
- Mass of neutron
  - \( M_n = 1.675 \times 10^{-27} \text{ kg} \)
- Mass of electron
  - \( M_e = 9.109 \times 10^{-31} \text{ kg} \)
- Using Einstein’s famous formula \( E = Mc^2 \), we can express mass in terms of energy (often done for subatomic particles)
  - ex. \( M_p = (1.673 \times 10^{-27} \text{ kg}) \times (2.998 \times 10^8 \text{ m/s})^2 = 1.504 \times 10^{-10} \text{ J} \)
    - \( M_p = 938.3 \text{ MeV (million eV)} \)
  - \( M_n = 939.6 \text{ MeV} \)
  - \( M_e = 0.511 \text{ MeV ( = 511 keV)} \)
  - Note that the mass of the proton and neutron are \( \sim 1837 \) times the mass of electron
Atomic nuclei

- Mass of atom

\[ M_A \approx ZM_H + NM_n - BE(A, Z, N) \]

where \( BE(A,Z,N) \) is the binding energy of the nucleus (function of \( A \), \( Z \), and \( N \)), and \( M_H \) is the mass of the hydrogen atom

- Binding energy is the energy released when protons and neutrons are brought together to form a nucleus

- For some elements, we can have naturally occurring stable (i.e. non radioactive) forms for different values of \( N \), hence \( A \)

- Nuclei with the same \( Z \) but different \( N \) are called isotopes
  - ex. hydrogen (\( Z=1 \)) has 2 stable isotopes with \( A=1 \) (\( N=0 \)) and \( A=2 \) (\( N=1 \))
  - ex. tin (Sn) (\( Z=50 \)) has 10 stable isotopes with
    \( A=112,114,115,116,117,118,119,120,122,124 \)

- Nuclei are often identified uniquely via the form \( ^A Z \) where the element name stands in for \( Z \) ex. \( ^{112}\text{Sn}, ^{114}\text{Sn}, ^{115}\text{Sn}, \ldots \)
Nuclear decay

• A nucleus may decay when there is a combination of products that have a smaller total energy
• Most common decay modes
  – $\gamma$, $\beta$, EC, $\alpha$, fission
• $\gamma$ decay
  – Emission of high energy photons in the transition from an excited nuclear state to a lower state. $A$, $Z$, $N$ of nucleus remains the same

\[ A, Z, N \]
Nuclear decay

- **β decay**
  - Emission of β particles (electrons) in the transition from one nuclear species to another
  - β particles can have negative charge (electrons) or positive charge (positrons)
  - Another particle always involved is the neutrino, ν – a chargeless, nearly massless, extremely weakly interacting particle
  - Typical energy releases are on the order of a few MeV
  - In β⁻ decay $A, Z, N \rightarrow A, Z+1, N-1 \Rightarrow$ a neutron changes into a proton

$A, Z, N$  

$A, Z+1, N-1$
Nuclear decay

- $\beta^+$ decay $A, Z, N \rightarrow A, Z-1, N+1 \Rightarrow$ a proton changes into a neutron

- Another process that completes with $\beta^+$ decay is *electron capture* (EC) where an atomic electron is captured by the nucleus
Nuclear decay

- **α decay**
  - Emission of a \(^4\)He nucleus
  - Typical energy release is 5 MeV
  - In α decay \(A, Z, N \rightarrow A-4, Z-2, N-2\) \(\Rightarrow\) nucleus loses 4 nucleons
    - 2 protons and 2 neutrons
Nuclear decay

- **Fission**
  - Nucleus breaks up into (typically) 2 lighter nuclei + some neutrons
  - Releases a large amount of energy (~190 MeV / fission)
  - Nuclear power reactors use fission of U
  - Nuclear weapons use fission of Pu (mainly) and U
Radioactive decay

- Probability that a radioactive nucleus will decay in a time $dt$ is $\lambda$ – the decay constant
- # of radioactive nuclei that decay in time $dt$ is
  \[
  \frac{dN}{dt} = -\lambda N
  \]
- Rearranging and solving for $N$
  \[
  \frac{dN}{N} = -\lambda dt \quad \Rightarrow \quad \int_{N_0}^{N} \frac{dn}{n} = -\int_{0}^{T} \lambda dt \quad \Rightarrow \quad \ln(N) - \ln(N_0) = -\lambda T
  \]
  \[
  N(T) = N_0 e^{-\lambda T}
  \]
- Where $N_0$ is the number of radioactive nuclei at time $T = 0$
Radioactive decay

• The *mean life* is defined as \( \tau = \frac{1}{\lambda} \)
  and at time \( T = \tau \), \( 1/e \) (36.8%) of the nuclei are left

• Since its hard to compute factors of \( e \) in our heads, we define a *half life* \( t_{1/2} = \ln(2) \times \tau \)

• At time \( T = t_{1/2} \), \( \frac{1}{2} \) of the original radioactive nuclei are left (\( \frac{1}{2} \) have decayed)

• After \( n \) half lives, only \( (\frac{1}{2})^n \) of the original nuclei are left

• *Activity* is defined as

\[
A(T) = \lambda N(T) = \lambda N_0 e^{-\lambda T} = \frac{dN}{dT}
\]

• Units are *Becquerel* (Bq) – 1 disintegration/s – or *Curies* (Ci) 
  1 Ci = \( 3.7 \times 10^{10} \) disintegrations/s