Absorption of Radiation

Lecture 28

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Review of L-26

• $\beta$’s loose energy in matter via 3 main processes
  – Ionization
  – Bremsstrahlung
  – Annihilation
• $\alpha$’s interact with matter mainly through ionization
• A quantity called *range* is defined as
  \[ \text{range} = \text{penetration depth} \times \text{density} \]
• For kinetic energy of $\beta$’s (or $e^-$) $> 0.6$ MeV, empirical relation
  \[ \text{Range} \left( \frac{\text{kg}}{\text{m}^2} \right) = 5.42E - 1.33 \]
  where $E$ is in MeV
• Distance of penetration $D$ for $\alpha$ particle of energy $E$ (MeV) *in air* is
  \[ D(\text{cm}) \approx 0.325E^{3/2} \]
Review of L-26

• Main processes for $\gamma$ rays interacting with matter
  – Coherent scattering
  – Compton scattering
  – Photoelectric effect
  – Pair production
  – Photonuclear reactions
Linear energy transfer (\textit{LET})

- Particles can lose energy continuously along their path when interacting with matter, and so continuously slow down.
- When working out the impact to biological systems (or matter), the amount of energy lost by the particle per unit distance, called the \textit{linear energy transfer}, or \textit{LET}, is important.

\[
\text{LET} = \frac{dE}{dx}
\]

- Particles with a large \textit{LET} can do a large amount of damage, but usually over a very short range.
- Particles with a small \textit{LET} do lesser damage, but can affect tissues further from the source.
- The energy of a particle is a nonlinear function of the distance travelled, so the \textit{LET} can vary enormously over a particles path.
- Very often the \textit{LET} is largest towards the \textit{end} of a particles path, so radiation treatments for cancer “tune” energies of, e.g. \textit{e}\textsuperscript{−} beams so that the \textit{e}\textsuperscript{−} stop in tumours.
Particles and $LET$

- Heavy charged particles and $\alpha$’s have a very high $LET$
- $e^\pm$ have a $LET$ that varies from small for high energies, to large for low energies
  - This is what makes $e$ beams useful for cancer therapies
  - Do minimal amount of damage to tissue at entry point, maximum damage to tumour
- Protons have a larger $LET$ than $e^-$, but smaller than $\alpha$’s
  - Also used in cancer therapies
- Other particles used in beam therapies for cancer treatment
  - Neutrons
  - Muons – like “heavy” electrons
  - Pions
  - Accelerated ions
- Each has its advantages/disadvantages
- By far most popular is X-ray beam therapy since every hospital has X-ray facility, but not always particle accelerator
Absorption of radiation

- Radiation classified in two main ways
  - Ionizing (charged particles, UV, X-rays, $\gamma$-rays, neutrons, …)
  - Non-ionizing (microwaves, radiowaves, …)
- Most damaging form of radiation are those causing ionization
- Ionizing radiation quantified by amount of ionization the radiation causes in dry air at NTP
- Physical processes that deposit energy in matter lead to ionization if an electron – positive-ion pair can be created
  - In dry air, this requires $\sim 35$ eV of energy (at NTP)
  - ex. An electron of energy 5 MeV is stopped in air at NTP, how many $e^-$-ion pairs are created?
    - $n = \frac{5 \times 10^6}{35} \approx 143000$ ion pairs
Exposure

• The strength of the radiation field is quantified by the exposure – the amount of radiation that will produce charge of either sign in dry air at NTP

• Exposure units are Coulombs/kg
  – Unit is Röntgen (R)
  – 1 R = 2.58 × 10^{-4} C/kg

• Ex. A radiation field causes the total production of 10^9 ion-pairs per kg of air
  – Exposure = 10^9 × 1.602 × 10^{-19} = 1.6 × 10^{-10} C/kg = 0.62 \mu R

• Very often, exposure is given in form of a rate, i.e. R/hr
  – If the same field now causes 10^9 ion pairs/s, 0.62 \mu R/s = 2.23 mR/hr
Radiation dose

- *Dose* = amount of energy absorbed per unit mass
  - SI unit Gray (Gy) = 1 J/kg
  - Old unit rad = 0.01 J/kg
- For charged particles, dose calculations straightforward (add up energy of all particles emitted divide by total mass of matter)
- For photons, more difficult (since photons are not slowed down and stopped by matter, but attenuated and scattered)
- Photon attenuation is (works for $\gamma$ and X and UV, ...)
  \[ I = I_0 e^{-\mu x} \]
  $\mu$ is attenuation coefficient
- Need energy absorption
Absorption coefficient

- Photon scattering – photon can leave volume without depositing all its energy
- *Absorption* coefficient takes into account photoelectric, pair, Compton absorption
- *Scattering* coefficient takes into account scattered photons
- Attenuation coefficient = absorption + scattering
- Very often, quantities expressed as mass attenuation, absorption,… coefficients

\[ \mu_m = \frac{\mu}{\rho} \quad \text{m}^2/\text{kg} \]

with \( \rho \) density
- For \( \gamma \) rays with \( 100 \text{ keV} < E_\gamma < 5 \text{ MeV} \), \( \mu_m \sim 3.0 \times 10^{-3} \text{ m}^2/\text{kg} \)
Absorbed dose

- For photons, dose \((D)\) is

\[
D \approx 3.0 \times 10^{-3} NEt \quad \text{J/kg}
\]

\(E\) is the photon energy (J), \(N\) is photon rate (#/s), \(t\) the time

- Ex. A 1 µCi source is ingested that is both a β and γ emitter, and spreads uniformly over the body of a 60 kg person. The average β energy is 1 MeV, and for each decay, a 1.5 MeV γ is emitted. What is the dose rate?
  - The β energy is totally absorbed
    \[
    = 1 \text{ MeV} \times 1 \times 10^{-6} \times 3.7 \times 10^{10} \\
    = 3.7 \times 10^4 \text{ MeV/s} \Rightarrow 617 \text{ MeV/kg/s} \text{ (divide by body mass)}
    \]
  - The γ energy absorbed
    \[
    D_\gamma \text{ s}^{-1} = 3.0 \times 10^{-3} \times 3.7 \times 10^4 \times 1.5 = 166.5 \quad \text{MeV/kg/s}
    \]
  - Dose rate = 45 µrad/hr
Biological half life

- Body processes ingested materials at specific rates, depending on the chemistry and metabolism
- Elimination of materials by body can be described by an exponential with a *biological* half life $t_{\frac{1}{2}}^b$
- Radionuclide ingested by body
  - Amount left after time $t$ is
    \[
    N_b(t) = N_0 e^{-\lambda t} e^{-\lambda_b t} = N_0 e^{-\lambda_{eff} t}
    \]
    where
    \[
    \lambda_{eff} = \lambda + \lambda_b = \frac{t_{\frac{1}{2}} + t_{\frac{1}{2}}^b}{t_{\frac{1}{2}} t_{\frac{1}{2}}^b}
    \]
Total dose

- Take our previous example, work out the total dose assuming a half life of 5 yr, and biological half life of 6 days
- \( \lambda_{\text{eff}} \approx \ln 2 / 6 \text{ d}^{-1} \)
- At time \( t \), the number of decays that occur in the body in interval \( dt \) is \( A \, dt \) (\( A \) is the activity = \( \lambda N \))
- Total number of decays in the body is

\[
\int_0^T A \, dt = \int_0^T \lambda N_0 e^{-\lambda_{\text{eff}} t} \, dt = \frac{\lambda}{\lambda_{\text{eff}}} \, N_0 \left( 1 - e^{-\lambda_{\text{eff}} T} \right)
\]

- With our example of a 1 \( \mu \text{Ci} \) source,
  \#decays = \( 37000 \times 6 \times 24 \times 3600 / \ln(2) = 2.77 \times 10^{10} \). and the total dose is \( 2.77 \times 10^{10} \times (1/60 (\beta \text{’s}) + 0.003(\gamma \text{’s})) \times 1.6 \times 10^{-13} \)
  \( = 8.72 \times 10^{-5} = 87 \mu \text{Gy} = 8.7 \text{ mrad} \)
Dose equivalent

- Different radiations have different consequences on biological tissues
- Take this into account by giving each type of radiation a weighting factor so that they can be compared to the equivalent dose of X-rays
  - Relative biological effectiveness (RBE) or radiation weighting factor ($w_R$) – these are just numbers
- Taking into account the weighting factor, we have the dose equivalent $H$
  - SI unit Sievert (Sv)
  - Old unit rem 100 rem = 1 Sv

$$H = \sum w_R \times D$$

So our previous example, $\beta$ and $\gamma$ have $w_R = 1$, so the dose equivalent is $87 \ \mu\text{Sv}$ or $8.7 \ \text{mrem}$