Rotational Motion and Moment of Inertia

**Purpose:** Measurements of the angular velocity of a flywheel permit calculation of its moment of inertia, which is compared with a theoretical calculation based on physical measurements.

**Reference:** “An Introduction to Error Analysis 2nd Edition” (Taylor, J.R.)
“Analytical Mechanics” (Fowles & Cassiday)

**Apparatus:** Flywheel and 1 kg mass
Rotational motion detector
Computer with Data Studio software installed

**Introduction:**

The apparatus consists of a flywheel connected to a rotational motion sensor, permitting the student to measure the flywheel’s angular velocity as a function of time. The flywheel axle is fitted with a pin permitting a cord to be wrapped around the axle. A hanging mass $m$ is attached by a cord to the axle of radius $r$. This provides a torque when the mass is dropped. As the mass falls, the flywheel experiences an angular acceleration $\alpha_1$. After several rotations of the flywheel the mass hits the floor, thus eliminating the torque due to the mass. However, a torque due to friction remains, and this torque eventually brings the flywheel to rest. As the flywheel slows down, it undergoes an angular acceleration $\alpha_2$, which is negative.

If $\tau_1$ and $\tau_2$ are the *magnitudes* of the torques due to the falling mass and friction, respectively, then during the deceleration phase

(1) \[ -\tau_2 = I\alpha_2 \]

and during the acceleration phase

(2) \[ \tau_1 - \tau_2 = I\alpha_1 \]

where $I$ is the moment of inertia of the flywheel and axle.
By considering the free-body diagram for the hanging mass \( m \) during the acceleration phase, and using equations (1) and (2), it is straightforward to show that:

\[
I = \frac{mr(g - \alpha_1 r)}{\alpha_1 - \alpha_2}
\]

The accelerations \( \alpha_1 \) and \( \alpha_2 \) are calculated from a graph of angular velocity versus time. The mass \( m \) will be provided in the laboratory, and you will measure the radius \( r \). Thus, an experimental value of the moment of inertia can be calculated.

Assuming that the flywheel and axle are of uniform density, the moment of inertia can also be calculated from geometric considerations. This value will be referred to as the theoretical value, and a comparison will be made between the experimental and theoretical values.

**Experiment:**

1. Turn on the computer and start the Data Studio acquisition software. Ensure that the computer is connected to the rotational motion sensor.

2. Open a new acquisition session, set it to record angular velocity as a function of time.

3. Wind the cord onto the axle of the flywheel and attach the mass to the free end of the cord. Press the “start” button on the acquisition software and drop the mass. When the flywheel comes to a complete stop, press the “stop” button.

4. Use the vernier calipers provided to measure the diameter of the axle, and the diameter and thickness of the disc portion of the flywheel. The mass of the flywheel (disc and axle) is provided, and the length of the axle (i.e., the shaft that protrudes to the left and right of the disc) is also provided. Remember to record appropriate error values.

**Analysis:**

1. Part of the introduction in your report should include the derivation of the expression for the moment of inertia \( I \) (Eq. 3) in terms of the quantities you measure. Begin the derivation of \( I \) with a free-body diagram, and then use Eq. 1 and Eq. 2.

2. From your graph of angular velocity, determine the values of the angular accelerations \( \alpha_1 \) and \( \alpha_2 \) by applying linear regression to the two linear ranges of your data. Export the data to a diskette for later analysis. Be selective in choosing the points to regress – for example, it is probably wise to ignore points that
correspond to obviously non-linear regions of the graph. A good way to do this is to plot the difference between the fitted straight line and the experimental data.

3. Create a histogram of the deviations \( y_i - A - Bx_i \) for the deceleration region. The histogram for the deceleration region may appear to be approaching a limiting normal distribution. Calculate the mean and standard deviation of the values \( y_i - A - Bx_i \) for the deceleration region. Determine the percentage of this data set within one and two standard deviations of the mean. Are the data points normally distributed? Why, or why not, is the use of the error formulas for the regression coefficients appropriate in this case? (ref. Taylor, section 8.3)

4. Using your expression from Eq. 3, you may now calculate an experimental value of \( I \) and its error. Compare your experimental value with the theoretical value. Are these values equal within experimental error?

5. Using your experimental value of \( I \) calculate the values of \( \tau_1 \) and \( \tau_2 \).

6. In your conclusions, summarize your findings. Try to comment on your results as much as possible. Do the experimental and theoretical values of \( I \) agree? Do the relative magnitudes of \( \tau_1 \) and \( \tau_2 \) make sense? In some sense, this experiment shows the validity of a very general physical law – namely \( \sum \tau = I \alpha \). How do your results demonstrate this?
WRITE-UP GUIDELINES FOR RESULTS SECTION OF ROTATIONAL MOTION AND MOMENT OF INERTIA EXPERIMENT

Your Results section on this report should contain the following:

1. A table showing your measurements of the apparatus and the results of your linear regressions.

2. Two graphs of angular velocity vs. time including linear regression lines for the acceleration and deceleration regions respectively.

3. Calculations of $I$ from the geometry of the apparatus and from the angular accelerations. Compare these results. Calculations of the applied torques.

4. Discussion of how calculations were done, how errors were estimated, etc.

5. The error propagation for the calculation of $I$ from the geometry of the apparatus is tricky and quite involved. It is highly recommended that you use Maple for it. Include your Maple program (with comments!) as a sample calculation. This should be the only sample calculation required for this lab.

Page limit (for entire report - Results and Conclusions): 10